Photon production from a thermalized quark gluon plasma: quantum kinetics and nonperturbative aspects

D. Boyanovsky[1,2,*] and H. J. de Vega[2,1,†]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*
[2]*LPTHE, Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII), Tour 16, 1er. étage, 4, Place Jussieu, 75252 Paris, Cedex 05, France*

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We study the production of photons from a quark gluon plasma in local thermal equilibrium by introducing a non-perturbative formulation of the real time evolution of the density matrix. The main ingredient is the real time effective action for the electromagnetic field to $\mathcal{O}(\alpha_{em})$ and to all orders in α_s. The real time evolution is completely determined by the solution of a *classical stochastic* non-local Langevin equation which provides a Dyson-like resummation of the perturbative expansion. The Langevin equation is solved in closed form by Laplace transform in terms of the thermal photon polarization. A quantum kinetic description emerges directly from this formulation. We find that photons with $k \lesssim 200$ Mev *thermalize* as plasmon quasiparticles in the plasma on time scales $t \sim 10 - 20$ fm/c which is of the order of the lifetime of the QGP expected at RHIC and LHC. We then obtain the direct photon yield to lowest order in α_{em} and to leading logarithmic order in α_s in a *uniform* expansion valid at all time. The yield during a QGP lifetime $t \sim 10$ fm/c is systematically larger than that obtained with the equilibrium formulation and the spectrum features a distinct flattening for $k \gtrsim 2.5$ Gev. We discuss the window of reliability of our results, the theoretical uncertainties in *any* treatment of photon emission from a QGP in LTE and the shortcomings of the customary S-matrix approach.

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I. INTRODUCTION

The quark gluon plasma (QGP) is a novel state of matter conjectured to be formed during ultrarelativistic heavy ion collisions and to have existed when the Universe was about 10 μsecs old. Ultrarelativistic heavy ion experiments at SPS-CERN, AGS-BNL, RHIC-BNL and the forthcoming LHC at CERN seek to create this state in collisions of heavy ions such as Pb and Au up to $\sqrt{s} \sim 200$ Gev/nucleon at RHIC and 5 Tev/nucleon at LHC . Current theoretical ideas suggest that a QGP in local thermodynamic equilibrium (LTE) is formed on a time scale of the order of ~ 1 fm/c after the collision when the partons in the colliding nuclei are liberated. Parton-parton collisions during a pre-equilibrium stage is then conjectured to lead to a state of local thermodynamic equilibrium that expands hydrodynamically and eventually undergoes a hadronization phase transition at a temperature of order ~ 160 Mev[1]. The experimental confirmation of a quark gluon plasma hinges on identifying observables that are directly associated with the properties of the QGP. Electromagnetic probes, namely photons and lepton pairs are considered to be "clean" since they only interact electromagnetically and their mean free paths are expected to be much larger than the size of the QGP. These probes are expected to leave the hot and dense region without further scattering, hence carrying direct information of the QGP[2, 3, 4]. These expectations led to an effort to obtain an assessment of direct photon emission and production of dilepton pairs from a thermalized QGP[2]-[13]. Preliminary assessments concluded that direct photon emission from a QGP in LTE can be larger than electromagnetic emission from a hadronic gas[4, 5].

The first observation of direct photon production in ultrarelativistic heavy ion collisions has been reported by the WA98 collaboration at SPS-CERN in $^{208}\mathrm{Pb} +^{208}\mathrm{Pb}$ at $\sqrt{s} = 158$ Gev/nucleon[14]. The WA98 data reveals an excess of direct photons above the expected background from hadronic decays in the range of transverse momentum $p_T > 1.5$ Gev in the most central collisions. These observations confirm the feasibility of direct photons as experimental probes for studying the formation and evolution of a QGP.

A variety of fits of the experimental data with various theoretical models has been reported[10], however the results are inconclusive: models with or without QGP emission seem to fit the data in a manner qualitatively not very different from the fits based solely on hadronic 'cocktails'[10].

*Electronic address: boyan@pitt.edu
†Electronic address: devega@lpthe.jussieu.fr

Photon data from ultrarelativistic heavy ion collisions at RHIC are forthcoming and we believe it is imperative to re-assess the current theoretical understanding of direct photon emission from a QGP in LTE for a reliable theoretical prediction, upon which may hinge the identification of a QGP.

The most widely used approach to study photon production from a QGP in LTE is based on the S-matrix calculation of an inclusive transition probability per unit space-time volume with initial asymptotic states of quarks and gluons averaged with a thermal distribution. Recent studies in real-time have provided a detailed criticism of this approach on the basis that the S-matrix calculation explicitly assumes an infinite lifetime for a QGP in LTE and treats the uncofined quarks and gluons in the plasma as initial asymptotic states[15, 16]. The underlying assumptions in the widely used S-matrix approach, namely, treating the medium as of infinite lifetime and spatial extent are manifestly inconsistent with the experimental and physical situation in which the state prior to the collision is described by hadrons and the ensuing QGP, if formed, only lasts during a (proper) time of order $\sim 10\text{fm}/c$ and its spatial scale is $\sim 10 fm$. There are preliminary studies of the influence of finite *size* effects on photon emission from the plasma[17, 18] and more recently for dilepton emission from the hadronic gas[19]. Studies of the finite *lifetime* effects on the photon production yield were reported in references[15, 16]. The results of these studies point out the importance of non-equilibrium real time processes whose contribution is subleading in the limit of infinite lifetime but that during the *finite* lifetime yield contributions of the same order of or larger than those found in the S-matrix approach.

Studying particle emission from a QGP is fundamentally different from a scattering experiment. In a scattering experiment the beam consists of physical particles, namely asymptotic "in" states, while in a QGP the quarks and gluons exist as a deconfined state of matter as a *transient* state. The "in"and "out" states are hadrons not quarks. The QGP thus emerges as an intermediate, transient state, and treating it as stationary source of electromagnetic radiation as is explicit in the S-matrix approach should be taken, at best, as an approximation.

The fundamentally correct calculation of photon production in an ultrarelativistic heavy ion collision must necessarily begin from an in state of nuclei formed by confined quark and gluons. These bound states evolve from the infinite past with the full Hamiltonian continuing the evolution through the collision and deconfinement process, through the thermalization and formation of a thermal QGP, through hadronization and eventually through final state interactions after freeze out. The photons produced during all of these processes will be measured as out states in the detector. Photons are produced at every stage, and the total number of photons *can* be calculated from the S-matrix if *all of the processes* are included in the dynamics with the inclusive out states being hadrons. Of course this is so far an unsolved theoretical task, and the problem emerges when one attempts to isolate individual stages of the dynamics and treat these stages, each of which has a finite time duration, in terms of an S-matrix calculation. For example, current estimates neglect the photons produced prior to the collision, treats the pre-equilibrium stage via parton cascade models, and ignores the photons from this stage in the calculation of emission from a QGP, which itself is treated as a steady state[15].

The potential identification of the formation and evolution of a QGP from electromagnetic probes from forthcoming RHIC data requires a reliable theoretical understanding of the phenomena.

In particular a recent detailed study of photon production in real and finite time from a QGP in LTE reveals that the spectrum of photons emitted during the finite lifetime of the QGP depends on the initial state at the time of thermalization and of the pre-equilibrium stage. This dependence is more pronounced for large momenta $k \gtrsim 4-5$ Gev. Such conclusion is in agreement with those obtained in ref.[13] wherein a sensitivity of the large k_T part of the spectrum to initial conditions was also found. The real time studies of photon production from a QGP in LTE with a finite lifetime in refs.[15, 16] all reveal a flattening of the spectrum at large momentum, the precise value of the momentum at which the spectrum flattens being sensitive to the pre-equilibrium stage which determines the initial state of the plasma at the thermalization hypersurface.

Perhaps coincidentally the WA98 data[14] displays a flattening of the spectrum for $p_T \gtrsim 1.5$ Gev.

Window of opportunity: Before a calculation of the direct photon yield from a QGP in LTE is attempted, it is important to establish the regime of experimental relevance and of theoretical reliability of *any* prediction of the yield based on LTE. The region of soft photons $k \lesssim 100-200$ Mev is experimentally complicated by the enormous background of photons from neutral pion decay (produced profusely in ultrarelativistic heavy ion collisions), bremsstrahlung from final state interactions etc. Assuming that LTE is established and maintained by quark and gluon collisions and assuming a collisional mean free path of order of $0.1 \lesssim \lambda \lesssim 0.3$ fm, photons with momenta $k \gtrsim 3-5$ Gev will likely probe scales shorter than the mean free path where the LTE approximation breaks down. Large departures from hydrodynamic behavior entailed by LTE is revealed in recent elliptic flow data on the parameter $v_2(p_T)$[20, 21] for $p_T > 2$ Gev. While this data indicates the breakdown of (ideal) hydrodynamics for hadrons, a similar breakdown is expected on physical grounds for hard electromagnetic probes when the Compton wavelength of the probe is smaller than the scale of the mean free path. Thus the reliability of a calculation of hard particle emission from a QGP in LTE must be re-assessed.

Furthermore the spectrum of high energy photons for $k \gtrsim 4-5$ Gev is sensitive to the initial conditions an the pre-equilibrium stage[13, 15]. Originally the pre-equilibrium stage was modelled as a parton cascade[22], however

more recently a different picture is emerging for the description of the pre-equilibrium stage based on color glass condensates[23]. Clearly the current understanding of the pre-equilibrium stage is still a matter of ongoing study.

Thus from the experimental point of view the relevant range of "clean" photons is probably for $k \gtrsim 100-200$ Mev, and from a theoretical point of view, the reliability of *any* calculation of the yield based on the assumption of LTE is probably suspect for $k \gtrsim 4-5$ Gev with the added uncertainty of dependence on the pre-equilibrium stage for momenta larger than this range. Therefore, in this article we will focus our study to the range 200 Mev $\lesssim k \lesssim 5$ Gev commenting on further uncertainties emerging from our study for the high energy region.

Goals of this article: In this article we study non-perturbative aspects of the real time dynamics of photon production and propagation with the goal of obtaining a deeper understanding of the direct photon yield during the finite lifetime of a transient QGP in LTE. In particular we focus on establishing a real time description of photon production and obtaining a theoretical prediction of the yield and the spectrum by including processes that are *not* included in the usual S-matrix calculation even at lowest order in α_{em}. These processes lead to subdominant contributions in the asymptotically long time limit, but their contribution during the finite lifetime of the QGP is of the same order of **or larger than** those extracted solely from an S-matrix analysis.

We begin by obtaining the time evolution of an initial density matrix directly from the effective action for the electromagnetic field *exact* to order α_{em} and to *all* orders in the strong coupling α_s. This formulation makes manifest the connection between photon production and the stochastic nature of photon emission and propagation in a thermalized plasma. The expression for the photon production yield obtained from this description reproduces the S-matrix results in a strict perturbative expansion in α_{em} for a steady state QGP. Moreover, when the full evolution is taken into account this formulation includes the dynamics of photon propagation in the medium and of thermalization. Furthermore, this formulation reproduces the results of kinetic theory, highlighting the limitations of the equilibrium approach to photon production.

Summary of main results:

- We provide a formulation of direct photon production in real time by obtaining the effective action for the electromagnetic field up to lowest order in α_{em} and in principle to all orders in α_s. Integrating out the quark and gluon fields to obtain the effective action leads to the description of the production and propagation of photons in a thermal bath. The properties of the QGP thermal bath are determined by current-current correlation functions and a stochastic gaussian colored noise that obey a generalized fluctuation-dissipation relation completely determined by the thermal photon polarization. The time evolution of the photon distribution function is determined by the solution of a *classical stochastic and non-local* Langevin equation. We explicitly solve the non-local Langevin equation in closed form by Laplace transform in terms of the thermal photon polarization. The resulting evolution represents a Dyson-like resummation of the naive perturbative expansion and provides a **uniform** expansion in α_{em} valid at all times and which includes non-perturbative aspects. We obtain the photon yield and the spectrum during the lifetime of the QGP expected at RHIC (≈ 10 fm/c) up to leading order in α_{em} and to logarithmic order in α_s.

- An important aspect that emerges from our study is that intermediate energy photons with $k \lesssim 400-500$ Mev propagate in the QGP plasma as *quasiparticles*, and in particular photons with $k \lesssim 200$ Mev *thermalize* with the plasma on time scales of the order of $\sim 10-20$ fm/c which is of the order of the lifetime of the QGP expected at RHIC.

- We predict the photon spectrum for the range of momenta 200 Mev $\lesssim k \lesssim 5$ Gev. The yield is calculated over a time scale compatible with the expected lifetime of a QGP at RHIC or LHC ~ 10 fm/c. In the intermediate region 200 Mev $\lesssim k \lesssim 2$ Gev the spectrum is similar to that obtained by the S-matrix formulation but systematically *larger* by a factor that ranges between $2-4$. The spectrum flattens at an energy scale $k \simeq 2.5$ Gev becoming dramatically *larger* than the yields obtained previously with the S-matrix formulation.

The article is organized as follows: in section II we obtain the real time effective action to lowest order in α_{em} and in principle to all orders in α_s. This effective action manifestly establishes contact with the stochastic nature of photon emission from a plasma in thermal equilibrium. In section III we obtain the photon distribution function in real time in terms of the solution of a Schwinger-Dyson equation which includes the photon self-energy to lowest order in α_{em} and in principle to all orders in α_s. This solution provides a Dyson-like resummation of the naive perturbative expansion. In section IV we show that the S-matrix result emerges in the strict perturbative limit. In this section we also make contact with a kinetic description of photon production which manifestly includes the dynamics of thermalization. In section V and VI we study the non-perturbative aspects and obtain the main results of this article. Section VII summarizes our conclusions and presents further questions.

II. THE REAL-TIME EFFECTIVE ACTION

In refs.[16, 24] a manifestly gauge invariant formulation of the time evolution of an initial density matrix has been described. We will follow this treatment as it guarantees that the results are completely gauge invariant. The gauge invariant Hamiltonian is given by[15, 16, 24]

$$H = H_{QCD}[\Psi] + \int d^3x \; \frac{1}{2} \; (\vec{E}_T^2 + \vec{B}^2) + e \int d^3x \; \mathbf{J} \cdot \mathbf{A}_T + H_{coul} \; , \tag{II.1}$$

where $H_{QCD}[\Psi]$ is the QCD Hamiltonian in absence of electromagnetism but in terms of the gauge invariant (under abelian gauge transformation) quark field(s) Ψ and the subscript T refers to transverse components. We have extended the fermion content to N_f flavors and the charge of each flavor species in units of the electron charge is included in the current, namely

$$\mathbf{J} = \sum_{i=1}^{N_f} \frac{e_i}{e} \; \bar{\Psi}_i \; \vec{\gamma} \; \Psi_i, \tag{II.2}$$

The instantaneous Coulomb interaction can be traded for a gauge invariant Lagrange multiplier field which we call A^0, (which, however should *not* be confused with the time component of the gauge field), leading to the following Lagrangian density

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{0,em} - J^0 A^0 + \mathbf{J} \cdot \mathbf{A}_T \quad ; \quad \mathcal{L}_{0,em} = \frac{1}{2} \left[(\partial_\mu \mathbf{A}_T)^2 + (\nabla A^0)^2 \right] \quad ; \quad J^\mu = \sum_{i=1}^{N_f} \frac{e_i}{e} \; \bar{\Psi}_i \; \gamma^\mu \; \Psi_i, \tag{II.3}$$

We will study the time evolution of the number of physical transverse photons and the expectation value of the (gauge invariant) transverse gauge field in the linearized approximation as an initial value problem.

In ref.[15] correlated initial states in which quark states are dressed by the electromagnetic field were studied in detail. One of the important conclusions of that study is that such an initial density matrix *cannot* correspond to a QGP in LTE under the strong interactions. This is a consequence of the fact that correlated quark-photon states necessarily require states constructed with the quark-current operator, which does not commute with H_{QCD}. Furthermore the results of this reference suggest that the details of the initial pre-equilibrium stage become manifest in the spectrum for $k \gtrsim 4$ Gev. The dependence of the high energy part of the spectrum on initial conditions has also been studied in[13] with similar conclusions.

Here we study the simpler case of an uncorrelated initial density matrix to highlight the non-perturbative aspects, with the understanding that a precise description of the high energy region will necessarily require a firmer knowledge of the initial state and the pre-equilibrium stage.

Hence we propose the initial density matrix to be of the form

$$\hat{\rho}(t_i) = \hat{\rho}_{QCD,i} \otimes \hat{\rho}_{\mathbf{A}_T,i} \tag{II.4}$$

The initial density matrix $\hat{\rho}_{QCD,i}$ describes a quark-gluon plasma in (local) thermodynamic equilibrium at a temperature $T = 1/\beta$, namely

$$\hat{\rho}_{QCD,i} = e^{-\beta \, H_{QCD}} \tag{II.5}$$

And $\hat{\rho}_{\mathbf{A}_T,i}$ is diagonal in the Fock representation of free field quanta of the transverse gauge field with an initial distribution of quanta. It is important to emphasize that any other initial density matrix that mixes photons and quarks will not commute with H_{QCD}[15], hence cannot describe a QGP in equilibrium.

The vacuum state is represented by $\hat{\rho}_{\mathbf{A}_T,i} = |0\rangle\langle 0|$ with $|0\rangle$ being the vacuum state of the field $\mathbf{A}_T(\vec{x})$.

In the field basis (Schroedinger representation) the matrix elements of $\hat{\rho}_{\mathbf{A}_T,i}$ are given by

$$\langle \mathbf{A}_T | \hat{\rho}_{\mathbf{A}_T,i} | \mathbf{A}_T^{'} \rangle = \hat{\rho}_{\mathbf{A}_T,i}(\mathbf{A}_T ; \mathbf{A}_T^{'}) \tag{II.6}$$

The time evolution of the initial density matrix is given by

$$\hat{\rho}(t_f) = e^{-iH(t_f - t_i)} \hat{\rho}(t_i) e^{iH(t_f - t_i)} \tag{II.7}$$

where H is the total Hamiltonian given by eq. (II.1). This particular choice of initial state will introduce transient evolution, however the long time behavior should be insensitive to this initial transient.

Furthermore, we point out that it is important to study the initial transient stage for the following reason. Photons are produced in the thermal bath since the initial density matrix does not commute with the total Hamiltonian (non-equilibrium) and they propagate in the medium as "quasiparticles". By studying the initial transient dynamics after photons and quarks are coupled we can address the question of the dynamics of the *formation* and propagation of the quasiparticle which will be studied in detail in section (VI).

Our goal is to obtain the real time evolution of the number of photons produced as well as that of the equation of motion for the expectation value of the (gauge invariant) transverse gauge field to lowest order in α_{em} but in principle to *all orders* in α_s. The equation of motion for the expectation value of the transverse gauge field will yield real time information on the formation and propagation of *transverse plasmon quasiparticles*(see section VI).

The strategy is to obtain the real-time effective action for the transverse gauge fields by integrating out the quark and gluon degrees of freedom to lowest order in α_{em} but to all orders in α_s. In this manner, the QGP in LTE is treated effectively as a thermal bath.

The calculation of correlation functions is facilitated by introducing currents coupled to the different fields. Furthermore since each time evolution operator in eqn. (II.7) will be represented as a path integral, we introduce different sources for the various fields for forward and backward time evolution operators, referred generically as J^+, J^- respectively. The forward and backward time evolution operators in presence of sources are $U(t_f, t_i; J^+)$, $U^{-1}(t_f, t_i, J^-)$ respectively.

In order to avoid cluttering of notation let us collectively denote by χ the quark and gluon fields which will be integrated out

In what follows we will ignore the Lagrange multiplier field A_0 since the Couloumb interaction will be irrelevant to leading order in α_{em}.

The non-equilibrium generating functional then is given by

$$\mathcal{Z}[j^+, j^-] = \mathrm{Tr} U(\infty, t_i; J^+)\hat{\rho}(t_i) U^{-1}(\infty, t_i, J^-) = \int D\mathbf{A}_{T,i} \int D\mathbf{A}'_{T,i}\, \rho_{\mathbf{A}_{T,i}}(\mathbf{A}_{T,i}; \mathbf{A}'_{T,i}) \int \mathcal{D}\mathbf{A}^{\pm}_T \int \mathcal{D}\chi^{\pm} e^{iS[\mathbf{A}^{\pm}_T, \chi^{\pm}; J^{\pm}_{\mathbf{A}_T}; J^{\pm}_{\chi}]} \tag{II.8}$$

where

$$S[\mathbf{A}^{\pm}_T, \chi^{\pm}; J^{\pm}_{\Phi}; J^{\pm}_{\chi}] = \int_{t_i}^{\infty} dt d^3x \left[\mathcal{L}_{0,em}(\mathbf{A}^+_T) + J^+_{\mathbf{A}}\mathbf{A}^+_T - \mathcal{L}_{0,em}(\mathbf{A}^-_T) - J^-_{\mathbf{A}}\mathbf{A}^-_T\right] + \int_{\mathcal{C}} d^4x \left[\mathcal{L}_{QCD}(\chi) + J\chi + e\mathbf{J}\cdot\mathbf{A}_T\right] \tag{II.9}$$

and $\mathcal{C}$ describes a contour in the complex time plane as follows: from t_i to $+\infty$ (forward) the fields and sources are $\mathbf{A}^+_T$, χ^+, J^+_{χ} (where χ collectively represents both quarks and gluons), from $+\infty$ back to t_i (backward) the fields and sources are $\mathbf{A}^-_T, \chi^-, J^-_{\chi}$ and from t_i to $t_i - i\beta$ (Euclidean or Matsubara) the fields and sources are $\mathbf{A}_T = 0; \chi^{\beta}, J^{\beta}_{\chi}$. Along the Euclidean branch the interaction term vanishes since the initial density matrix for the quark and gluon fields, generically denoted as χ is assumed to be that of a QGP in thermal equilibrium. One can in principle consider a correlated initial state of a QGP and electromagnetic fluctuations but such state *does not* describe a QGP in thermal equilibrium and such initial density matrix would not commute with H_{QCD}, contrary to the usual assumption of an equilibrated QGP[15].

We seek to obtain the real time effective action for the transverse gauge field. Therefore we integrate out (trace over) the quark and gluon fields. After carrying out the trace over the QCD degrees of freedom, the remaining path integrals over $\mathbf{A}_T$ with the real time effective action have boundary conditions on the field $\mathbf{A}_T$ given by

$$\mathbf{A}^+_T(\vec{x}, t = t_i) = \mathbf{A}_{T,i}(\vec{x}) \quad ; \quad \mathbf{A}^-_T(\vec{x}, t = t_i) = \mathbf{A}'_{T,i}(\vec{x}) \tag{II.10}$$

The real time effective action for $\mathbf{A}_T$ is obtained by treating the quark and gluon fields as a bath performing the path integral over the quark and gluon degrees of freedom, namely, tracing over the bath degrees of freedom, leads to the influence functional[25] for $\mathbf{A}^{\pm}_T$.

The initial density matrix for the $\mathbf{A}_T$ field will be specified later as part of the initial value problem.

To lowest order in α_{em} but to *all* orders in α_s the real time effective action for $\mathbf{A}_T$ is obtained as follows. Insofar as the path integral over the QCD degrees of freedom is concerned, $\mathbf{A}_T$ is simply a background field, hence

$$\int \mathcal{D}\chi^{\pm} e^{i\int_{\mathcal{C}} \mathcal{L}_{QCD}(\chi^{\pm}) + e\mathbf{J}\cdot\mathbf{A}_T} \equiv \langle e^{i\int_{\mathcal{C}} e\mathbf{J}\cdot\mathbf{A}_T}\rangle_{QCD} \tag{II.11}$$

expanding the expectation value in powers of e

$$\langle e^{i\int_{\mathcal{C}} e\mathbf{J}\cdot\mathbf{A}_T}\rangle_{QCD} = 1 + ie\int_{\mathcal{C}}\langle\mathbf{J}\rangle_{QCD}\cdot\mathbf{A}_T + \frac{(ie)^2}{2}\int_{\mathcal{C}}\int_{\mathcal{C}}\langle\mathbf{J}^i\mathbf{J}^j\rangle_{QCD}\mathbf{A}^i_T\mathbf{A}^j_T + \mathcal{O}(e^3) = e^{-\frac{e^2}{2}\int_{\mathcal{C}}\int_{\mathcal{C}}\langle\mathbf{J}^i\mathbf{J}^j\rangle_{QCD}\mathbf{A}^i_T\mathbf{A}^j_T} + \mathcal{O}(e^3) \tag{II.12}$$

where we have used $\langle\mathbf{J}\rangle_{QCD} = 0$ and the connected current-current correlation function $\langle\mathbf{J}^i\mathbf{J}^j\rangle_{QCD}$ is in principle to all orders in α_s.

We introduce the spatial Fourier transform of the quark current $\mathbf{J}$ as

$$\mathbf{j}(\vec{k};t) = \frac{1}{\sqrt{\Omega}}\int d^3x e^{i\vec{k}\cdot\vec{x}}\mathbf{J}(\vec{x},t) \tag{II.13}$$

with Ω the quantization volume, in terms of which we obtain following the correlation functions

$$e^2\langle\mathbf{j}_l(\vec{k};t)\mathbf{j}_m(-\vec{k};t')\rangle = e^2\langle\mathbf{j}^-_l(\vec{k};t)\mathbf{j}^+_m(-\vec{k};t')\rangle = \Sigma^>_{lm}(\vec{k};t-t') = \Sigma^{-+}_{lm}(\vec{k};t,t') \tag{II.14}$$

$$e^2\langle\mathbf{j}_m(-\vec{k};t')\mathbf{j}_l(\vec{k};t)\rangle = e^2\langle\mathbf{j}^+_l(\vec{k};t)\mathbf{j}^-_m(-\vec{k};t')\rangle = \Sigma^<_{lm}(\vec{k};t-t') = \Sigma^{+-}_{lm}(\vec{k};t,t') = \Sigma^{-+}_{ml}(\vec{k};t',t) \tag{II.15}$$

$$e^2\langle T\ \mathbf{j}_l(\vec{k};t)\mathbf{j}_m(-\vec{k};t')\rangle = \Sigma^>_{lm}(\vec{k};t-t')\Theta(t-t') + \Sigma^<_{lm}(\vec{k};t-t')\Theta(t'-t) = \Sigma^{++}_{lm}(\vec{k};t,t') \tag{II.16}$$

$$e^2\langle\tilde{T}\ \mathbf{j}_l(\vec{k};t)\mathbf{j}_m(-\vec{k};t')\rangle = \Sigma^>_{lm}(\vec{k};t-t')\Theta(t'-t) + \Sigma^<_{lm}(\vec{k};t-t')\Theta(t-t') = \Sigma^{--}_{lm}(\vec{k};t,t') \tag{II.17}$$

where the expectation values are in the equilibrium thermal density matrix of the QGP, which results in translational and rotational invariant correlation functions that are only functions of the time differences and the superscripts $\pm$ refer to the forward (+) or backward (−) time branches. The symbols T and $\tilde{T}$ refer to time and antitime ordering respectively. These correlation functions are not independent, they obey

$$\Sigma^{++}_{lm}(\vec{k};t,t') + \Sigma^{--}_{lm}(\vec{k};t,t') - \Sigma^{-+}_{lm}(\vec{k};t,t') - \Sigma^{+-}_{lm}(\vec{k};t,t') = 0 \tag{II.18}$$

It is convenient at this stage to separate the transverse and longitudinal components of the polarization tensor $\Sigma_{lm}(\vec{k};t-t')$ as follows

$$\Sigma_{lm}(\vec{k};t-t') = \mathcal{P}_{lm}(\hat{\mathbf{k}})\Sigma_T(\vec{k};t-t') + \hat{\mathbf{k}}_l\hat{\mathbf{k}}_m\Sigma_L(\vec{k};t-t') \tag{II.19}$$

$$\mathcal{P}_{lm}(\hat{\mathbf{k}}) = \delta_{lm} - \hat{\mathbf{k}}_l\hat{\mathbf{k}}_m \tag{II.20}$$

The non-equilibrium real time effective action in terms of the spatial Fourier transforms of the fields and correlation functions to lowest order in α_{em} but to *all orders* in α_s is therefore given by

$$\begin{aligned} iS_{eff}[\mathbf{A}^+_T,\mathbf{A}^-_T] = \sum_{\vec{k}}\Big\{&\frac{i}{2}\int dt\Big[\dot{\mathbf{A}}^+_{\vec{k},T}(t)\cdot\dot{\mathbf{A}}^+_{-\vec{k},T}(t) - k^2\mathbf{A}^+_{\vec{k},T}(t)\cdot\mathbf{A}^+_{-\vec{k},T}(t) \\ &-\dot{\mathbf{A}}^-_{\vec{k},T}(t)\dot{\mathbf{A}}^-_{-\vec{k},T}(t) + k^2\mathbf{A}^-_{\vec{k},T}(t)\mathbf{A}^-_{-\vec{k},T}(t)\Big] \\ &-\frac{1}{2}\int dt\int dt'\Big[\mathbf{A}^+_{\vec{k},T}(t)\cdot\mathbf{A}^+_{-\vec{k},T}(t')\Sigma^{++}_T(\vec{k};t-t') + \mathbf{A}^-_{\vec{k},T}(t)\cdot\mathbf{A}^-_{-\vec{k},T}(t')\Sigma^{--}_T(\vec{k};t-t') \\ &-\mathbf{A}^+_{\vec{k},T}(t)\cdot\mathbf{A}^-_{-\vec{k},T}(t')\Sigma^{+-}_T(\vec{k};t-t') - \mathbf{A}^-_{\vec{k},T}(t)\cdot\mathbf{A}^+_{-\vec{k},T}(t')\Sigma^{-+}_T(\vec{k};t-t')\Big]\Big\} \end{aligned} \tag{II.21}$$

As it will become clear below, it is more convenient to introduce the following Wigner center of mass and relative variables for the transverse gauge field

$$\vec{\mathcal{A}}(\vec{x},t) = \frac{1}{2}\left(\mathbf{A}^+_T(\vec{x},t) + \mathbf{A}^-_T(\vec{x},t)\right) \ ; \ \mathbf{a}(\vec{x},t) = \mathbf{A}^+_T(\vec{x},t) - \mathbf{A}^-_T(\vec{x},t) \tag{II.22}$$

and the Wigner transform of the initial density matrix for the transverse gauge field

$$\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i) = \int D\vec{a}_i\ e^{-i\int d^3x\vec{\mathcal{E}}_i(\vec{x})\cdot\vec{a}_i(\vec{x})}\rho(\vec{\mathcal{A}}_i + \frac{\vec{a}_i}{2};\vec{\mathcal{A}}_i - \frac{\vec{a}_i}{2}) \ ; \ \rho(\vec{\mathcal{A}}_i + \frac{\vec{a}_i}{2};\vec{\mathcal{A}}_i - \frac{\vec{a}_i}{2}) = \int D\vec{\mathcal{E}}_i\ e^{i\int d^3x\vec{\mathcal{E}}_i(\vec{x})\cdot\vec{a}_i(\vec{x})}\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i) \tag{II.23}$$

The initial conditions on the $\mathbf{A}_T$ path integral given by (II.10) translate into the following initial conditions on the center of mass and relative variables

$$\vec{\mathcal{A}}(\vec{x},t=0)=\vec{\mathcal{A}}(\vec{x}) \;\; ; \;\; \mathbf{a}(\vec{x},t=0)=\mathbf{a}_i \tag{II.24}$$

The center of mass variable plays an important role: its expectation value is the mean field, since the expectation values of $\vec{A}^{\pm}_T(\vec{x},t)$ coincide,

$$\langle \vec{A}^{+}_T(\vec{x},t)\rangle = Tr\vec{A}_T(\vec{x},t)\;\rho = Tr\rho\;\vec{A}_T(\vec{x},t)=\langle \vec{A}^{-}_T(\vec{x},t)\rangle \tag{II.25}$$

In terms of the spatial Fourier transforms of the center of mass and relative variables (II.22) introduced above, integrating by parts and accounting for the boundary conditions (II.24) the non-equilibrium effective action (II.21) becomes:

$$\begin{aligned} iS_{eff}[\vec{\mathcal{A}},\mathbf{a}] &= \int dt \sum_{\vec{k}} \left\{-i\;\vec{a}_{-\vec{k}}(t)\cdot\left(\ddot{\vec{\mathcal{A}}}_{\vec{k}}(t)+k^2\vec{\mathcal{A}}_{\vec{k}}(t)\right)\right\} \\ &- \int dt\int dt' \left\{\frac{1}{2}\vec{a}_{-\vec{k}}(t)\cdot\vec{a}_{\vec{k}}(t')\mathcal{K}_T(k;t-t')+\vec{a}_{-\vec{k}}(t)\cdot\vec{\mathcal{A}}_{\vec{k}}(t')\;i\Sigma^R_T(k;t-t')\right\} \\ &+ \int d^3x\vec{a}_i(\vec{x})\cdot\dot{\vec{\mathcal{A}}}(\vec{x},t=0) \end{aligned} \tag{II.26}$$

where the last term arises after the integration by parts in time, using the boundary condition (II.24). There is no contribution from the $t\to\infty$ limit since the fields $\vec{\mathcal{A}}_{\vec{k}}(t)$ at non-zero temperature will vanish at asymptotically long time. The kernels in the above effective Lagrangian are given by

$$\mathcal{K}_T(k;t-t') = \frac{1}{2}\left[\Sigma^{>}_T(k;t-t')+\Sigma^{<}_T(k;t-t')\right] \tag{II.27}$$

$$i\Sigma^R_T(k;t-t') = \left[\Sigma^{>}_T(k;t-t')-\Sigma^{<}_T(k;t-t')\right]\Theta(t-t')\equiv i\Sigma_T(k;t-t')\Theta(t-t') \tag{II.28}$$

where we have used the fact that the polarization is a function of the modulus of the wavevector by rotational invariance. The photon polarization is computed to lowest order in α_{em} and in principle to all orders in α_s.

The quadratic part in $e^{iS_{eff}[\vec{\mathcal{A}},\mathbf{a}]}$ in the relative variable $\vec{a}$ can be written in terms of a stochastic noise variable ξ as

$$\begin{aligned} \exp\left\{-\frac{1}{2}\int dt\int dt'\vec{a}_{-\vec{k}}(t)\cdot\vec{a}_{\vec{k}}(t')\mathcal{K}_T(k;t-t')\right\} = \int\mathcal{D}\vec{\xi}\exp\Big\{-\frac{1}{2}\int dt\int dt'\;\;\xi_{\vec{k},l}(t)\mathcal{K}^{-1}_T(t-t')\xi_{-\vec{k},l}(t') \\ +i\int dt\;\;\xi_{-\vec{k},l}(t)\cdot a_{\vec{k},l}(t)\Big\} \end{aligned} \tag{II.29}$$

The non-equilibrium generating functional can now be written as

$$\begin{aligned} \mathcal{Z} = \int D\vec{\mathcal{A}}_i\int D\vec{\mathcal{E}}_i\int \mathcal{D}\vec{\mathcal{A}}\mathcal{D}\vec{a}\mathcal{D}\vec{\xi}\;\;\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)D\;\vec{a}_i e^{i\int d^3x\vec{a}_i(\vec{x})\cdot\left(\vec{\mathcal{E}}_i(\vec{x})-\dot{\vec{\mathcal{A}}}(\vec{x},t=0)\right)}\mathcal{P}[\xi] \\ \exp\left\{-i\int dt\sum_{\vec{k}}\;a_{-\vec{k},l}(t)\left[\ddot{\mathcal{A}}_{\vec{k},l}(t)+k^2\mathcal{A}_{\vec{k},l}(t)+\int dt'\;\;\Sigma^R_T(k;t-t')\mathcal{A}_{\vec{k},l}(t')-\xi_{\vec{k},l}(t)\right]\right\} \end{aligned} \tag{II.30}$$

where the noise probability distribution function is given by

$$\mathcal{P}[\vec{\xi}]=\exp\left\{-\frac{1}{2}\int dt\int dt'\sum_{\vec{k}}\;\;\xi_{\vec{k},l}(t)\mathcal{K}^{-1}_T(k;t-t')\xi_{-\vec{k},l}(t')\right\} \tag{II.31}$$

The functional integral over $\vec{a}_i$ can now be done, resulting in a functional delta function, that fixes the initial condition $\dot{\mathcal{A}}_T(\vec{x},t=0)=\mathcal{A}_{T,i}(\vec{x})$.

Finally the path integral over the relative variable $\vec{a}$ can be performed leading to a functional delta function and the final form of the generating functional is given by

$$\mathcal{Z}=\int D\vec{\mathcal{A}}_iD\vec{\mathcal{E}}_i\;\;\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)\mathcal{D}\vec{\mathcal{A}}\mathcal{D}\xi\;\;\mathcal{P}[\xi]\;\Pi_{\vec{k},l}\delta\left[\ddot{\mathcal{A}}_{\vec{k},l}(t)+k^2\mathcal{A}_{\vec{k},l}(t)+\int_0^t dt'\;\;\Sigma_T(k;t-t')\mathcal{A}_{\vec{k},l}(t')-\xi_{\vec{k},l}(t)\right] \tag{II.32}$$

with the initial conditions on the path integral on Ψ given by

$$\vec{\mathcal{A}}(\vec{x},t=0)=\vec{\mathcal{A}}_i(\vec{x}) \;\; ; \;\; \dot{\vec{\mathcal{A}}}(\vec{x},t=0)=\vec{\mathcal{E}}_i(\vec{x}) \tag{II.33}$$

and we have used the definition of $\Sigma^R_T(k;t-t')$ in terms of $\Sigma_T(k;t-t')$ given in equation (II.28).

The meaning of the above generating functional is the following: in order to obtain the correlation functions of the center of mass Wigner variable $\vec{\mathcal{A}}$ we must first find the solution of the *classical stochastic* non-local Langevin equation of motion

$$\begin{aligned}
&\ddot{\mathcal{A}}_{\vec{k},l}(t)+k^2\mathcal{A}_{\vec{k},l}(t)+\int_0^t dt' \;\; \Sigma_T(k;t-t')\mathcal{A}_{\vec{k},l}(t')=\xi_{\vec{k},l}(t)\\
&\vec{\mathcal{A}}(\vec{x},t=0)=\vec{\mathcal{A}}_i(\vec{x}) \;\; ; \;\; \dot{\vec{\mathcal{A}}}(\vec{x},t=0)=\vec{\mathcal{E}}_i(\vec{x})
\end{aligned} \tag{II.34}$$

for arbitrary noise term ξ and then average the products of $\vec{\mathcal{A}}[\xi]$ over the stochastic noise with the probability distribution $\mathcal{P}[\xi]$ given by (II.30), and finally average over the initial configurations $\vec{\mathcal{A}}_i(\vec{x})$; $\vec{\mathcal{E}}_i(\vec{x})$ weighted by the Wigner function $\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)$, which plays the role of an initial semiclassical phase space distribution function.

Calling the solution of (II.34) $\vec{\mathcal{A}}_{\vec{k},l}(t;\vec{\xi};\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)$, the two point correlation function, for example, is given by

$$\begin{aligned}
\langle\vec{\mathcal{A}}_{-\vec{k},l}(t)\vec{\mathcal{A}}_{\vec{k},l}(t')\rangle = \;\; & \mathcal{Z}^{-1}\int\mathcal{D}[\xi]\mathcal{P}[\xi]\int D\vec{\mathcal{A}}_i\int D\vec{\mathcal{E}}_i \;\; \mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)\; \vec{\mathcal{A}}_{-\vec{k},l}(t;\vec{\xi};\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)\,\vec{\mathcal{A}}_{\vec{k},l}(t';\vec{\xi};\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)\\
&\times\delta\left[\ddot{\mathcal{A}}_{\vec{k},l}(t)+k^2\mathcal{A}_{\vec{k},l}(t)+\int_0^t dt' \;\; \Sigma_T(k;t-t')\mathcal{A}_{\vec{k},l}(t')-\xi_{\vec{k},l}(t)\right]
\end{aligned} \tag{II.35}$$

We note that in computing the averages and using the functional delta function to constrain the configurations of $\vec{\mathcal{A}}$ to the solutions of the Langevin equation, there is the Jacobian of the operator

$$\delta(t-t')\left[\frac{d^2}{dt^2}+k^2\right]+\Sigma^R_T(k;t-t') \tag{II.36}$$

which, however, is independent of the field and cancels between numerator and denominator in the averages.

This formulation establishes the connection with a *stochastic* problem and is similar to the Martin-Siggia-Rose[26] path integral formulation for stochastic phenomena. There are two different averages:

- The average over the stochastic noise term, which up to this order is Gaussian. We denote the averages over the noise with the probability distribution function $P[\xi]$ given by eqn. (II.31) as

  $$\langle\langle\mathcal{O}[\xi]\rangle\rangle\equiv\frac{\int\mathcal{D}\xi P[\xi]\mathcal{O}[\xi]}{\int\mathcal{D}\xi P[\xi]}. \tag{II.37}$$

  Since the noise probability distribution function is Gaussian the only necessary correlation functions for the noise are given by

  $$\langle\langle\xi_{\vec{k},l}(t)\rangle\rangle=0\;,\;\langle\langle\xi_{\vec{k},l}(t)\xi_{\vec{k}',j}(t')\rangle\rangle=\mathcal{P}_{lj}(\hat{\mathbf{k}})\;\mathcal{K}_T(k;t-t')\delta^3(\vec{k}+\vec{k}') \tag{II.38}$$

  and the higher order correlation functions are obtained from Wick's theorem.

- Average over the initial conditions with the Wigner distribution function $\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)$ which we denote as

  $$\overline{\mathcal{O}[\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i]}\equiv\frac{\int D\vec{\mathcal{A}}_i\int D\vec{\mathcal{E}}_i \;\; \mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)\mathcal{O}[\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i]}{\int D\vec{\mathcal{A}}_i\int D\vec{\mathcal{E}}_i \;\; \mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)} \tag{II.39}$$

  The initial Wigner distribution function requires precise knowledge of the pre-equilibrium stage. It allows to include initial correlations of quarks and photons, namely "entangled states" but as discussed in detail in ref.[15]

such initial density matrix will not commute with the H_{QCD} Hamiltonian and will not describe a state in LTE under the strong interactions. All of the theoretical uncertainties about the initial state prior to equilibration are encoded in the initial density matrix or alternatively in the initial Wigner distribution function $\mathcal{W}(\vec{\mathcal{A}}_i;\vec{\mathcal{E}}_i)$. In ref.[15] a particular state with initial correlations reflecting a pre-equilibrium stage was modelled. While we can consider such modelled initial state, in this article we focus on extracting non-perturbative aspects in the simplest and cleanest scenario, that of an initially uncorrelated Gaussian state for photons. The numerical study performed in ref.[15] revealed that initial state preparation on time scales of $\mathcal{O}(1\ \mathrm{fm}/c)$ modifies the photon spectrum for hard momenta $k \gtrsim 4$ Gev. This is roughly the scale of momenta at which the assumption of LTE breaks down in any case because the photon probes scales of the order of the mean free path for quark-gluon collisions. Thus studying an uncorrelated initial state, which is compatible with all previous S-matrix calculations is phenomenologically relevant for photon momenta $k \leq 4-5$ Gev.

Therefore in what follows we will consider a Gaussian initial Wigner distribution function with the following averages:

$$\overline{\mathcal{A}^a_{i,\vec{k}}\mathcal{A}^b_{i,-\vec{k}}} = \frac{\mathcal{P}^{ab}(\hat{\mathbf{k}})}{2k}\left[1+2\mathcal{N}_k\right] + \overline{\mathcal{A}^a_{i,\vec{k}}}\ \ \overline{\mathcal{A}^b_{i,-\vec{k}}}\ ; \tag{II.40}$$

$$\overline{\mathcal{E}^a_{i,\vec{k}}\mathcal{E}^b_{i,-\vec{k}}} = \mathcal{P}^{ab}(\hat{\mathbf{k}})\frac{k}{2}\left[1+2\mathcal{N}_k\right] + \overline{\mathcal{E}^a_{i,\vec{k}}}\ \ \overline{\mathcal{E}^b_{i,-\vec{k}}}\ ; \tag{II.41}$$

$$\overline{\mathcal{A}^a_{i,\vec{k}}\mathcal{E}^b_{i,-\vec{k}} + \mathcal{E}^b_{i,-\vec{k}}\mathcal{A}^a_{i,\vec{k}}} = \overline{\mathcal{A}^a_{i,\vec{k}}}\ \ \overline{\mathcal{E}^b_{i,-\vec{k}}} + \mathrm{c.c} \tag{II.42}$$

where $\mathcal{N}_k$ is the initial distribution of photons. The indices a, b refer to vector components, while the label i refers to the initial conditions (II.33).

Thus averages in the time evolved full density matrix are therefore given by

$$\langle \cdots \rangle = \overline{\langle\langle \cdots \rangle\rangle}\ . \tag{II.43}$$

The noise field emerged as an auxiliary variable to represent the quadratic contribution from the relative variable in eqn. (II.26). While the full microscopic dynamics is completely Hamiltonian and therefore deterministic, integrating out the QGP degrees of freedom leads to a reduced density matrix and the ensuing coarse grained dynamics for the photon field, just as in the microscopic treatment of Brownian motion[25].

A. Relation between Σ and $\mathcal{K}$: Fluctuation and Dissipation

From the expression (II.28) for the photon polarization which is determined by averages in the equilibrium density matrix of QCD we now obtain a dispersive representation for the kernels $\mathcal{K}_T(k;t-t')$ and $\Sigma^R_T(k;t-t')$. This is achieved by explicitly writing the expectation value in terms of energy eigenstates of the bath (quark and gluon fields) introducing the identity in this basis and using the time evolution of the Heisenberg field operators to obtain

$$\Sigma^>_T(k;t-t') = \int_{-\infty}^{\infty} d\omega\ \sigma^>_T(k;\omega)\ e^{i\omega(t-t')} \tag{II.44}$$

$$\Sigma^<_T(k;t-t') = \int_{-\infty}^{\infty} d\omega\ \sigma^<_T(k;\omega)\ e^{i\omega(t-t')} \tag{II.45}$$

with the spectral functions

$$\sigma^>_T(k;\omega) = e^2\frac{\mathcal{P}_{ij}(\hat{\mathbf{k}})}{2Z_{QCD}}\sum_{m,n} e^{-\beta E_n}\langle n|\mathbf{j}_i(\vec{k},0)|m\rangle\langle m|\mathbf{j}_j(-\vec{k},0)|n\rangle\,\delta(\omega-(E_n-E_m)) \tag{II.46}$$

$$\sigma^<_T(k;\omega) = e^2\frac{\mathcal{P}_{ij}(\hat{\mathbf{k}})}{2Z_{QCD}}\sum_{m,n} e^{-\beta E_n}\langle n|\mathbf{j}_j(-\vec{k},0)|m\rangle\langle m|\mathbf{j}_i(\vec{k},0)|n\rangle\,\delta(\omega-(E_m-E_n)) \tag{II.47}$$

where Z_{QCD} is the QCD equilibrium partition function, the states $|n>$ are exact eigenstates of H_{QCD} and we have used rotational invariance. Upon relabelling $m \leftrightarrow n$ in the sum in eq. (II.47) we find the KMS relation[27, 28]

$$\sigma^<_T(k;\omega) = \sigma^>_T(k;-\omega) = e^{\beta\omega}\sigma^>_T(k;\omega) \tag{II.48}$$

where we have used parity and rotational invariance in the second line above to assume that the spectral functions only depend of the absolute value of the momentum.

Using the spectral representation of the $\Theta(t-t')$ we find the following representation for the retarded photon polarization given by eqn. (II.28)

$$\Sigma^R_T(k;t-t') = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}e^{ik_0(t-t')}\tilde{\Sigma}^R_T(k,k_0) \tag{II.49}$$

with

$$\tilde{\Sigma}^R_T(k,k_0) = \int_{-\infty}^{\infty}d\omega\frac{[\sigma^>_k(\omega)-\sigma^<_k(\omega)]}{\omega-k_0+i\epsilon} \tag{II.50}$$

Using the condition (II.48) the above spectral representation can be written in a more useful manner as

$$\tilde{\Sigma}^R_T(k,k_0) = -\frac{1}{\pi}\int_{-\infty}^{\infty}d\omega\frac{\mathrm{Im}\tilde{\Sigma}^R_T(k,\omega)}{\omega-k_0+i\epsilon} \tag{II.51}$$

$$\mathrm{Im}\tilde{\Sigma}^R_T(k,\omega) = \pi\sigma^>_T(k;\omega)\left[e^{\beta\omega}-1\right] \tag{II.52}$$

clearly $\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega>0)>0$. Eq. (II.48) entails that the imaginary part of the retarded photon polarization is an odd function of frequency, namely

$$\mathrm{Im}\tilde{\Sigma}^R_T(k,\omega) = -\mathrm{Im}\tilde{\Sigma}^R_T(k,-\omega) \tag{II.53}$$

which is manifest in eq. (II.52).

The relation (II.52) leads to the following results which will prove to be useful later

$$\sigma^>_T(k;\omega) = \frac{1}{\pi}\mathrm{Im}\tilde{\Sigma}^R_T(k,\omega)\,n(\omega) \tag{II.54}$$

$$\sigma^<_T(k;\omega) = \frac{1}{\pi}\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\,\left[1+n(\omega)\right] \tag{II.55}$$

Similarly from the eqs. (II.27) and (II.44), (II.45) and the condition (II.48) we find

$$\mathcal{K}_T(k;t-t') = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}e^{ik_0(t-t')}\tilde{\mathcal{K}}_T(k;k_0) \tag{II.56}$$

$$\tilde{\mathcal{K}}_T(k;k_0) = \pi\sigma^>_T(k;k_0)\left[e^{\beta k_0}+1\right] \tag{II.57}$$

whereupon using the condition (II.48) leads to the generalized form of the fluctuation-dissipation relation

$$\tilde{\mathcal{K}}_T(k;k_0) = \mathrm{Im}\tilde{\Sigma}^R_T(k;k_0)\coth\left[\frac{\beta k_0}{2}\right] \tag{II.58}$$

Thus we see that $\mathrm{Im}\tilde{\Sigma}^R_T(k;k_0)\,;\,\tilde{\mathcal{K}}_T(k;k_0)$ are odd and even functions of the frequency respectively.

For further analysis below we will also need the following representation for $\Sigma_T(k;t-t')$ introduced in eq. (II.28)

$$\Sigma_T(k;t-t') = -i\int_{-\infty}^{\infty}e^{i\omega(t-t')}\left[\sigma^>_T(k;\omega)-\sigma^<_T(k;\omega)\right]d\omega = \frac{i}{\pi}\int_{-\infty}^{\infty}e^{i\omega(t-t')}\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)d\omega \tag{II.59}$$

whose Laplace transform is given by

$$\tilde{\Sigma}_T(k;s) \equiv \int_0^\infty dte^{-st}\Sigma_T(k;t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)}{\omega+is}d\omega \tag{II.60}$$

This spectral representation, combined with (II.51) lead to the relation

$$\tilde{\Sigma}^R_T(k;\omega) = \tilde{\Sigma}_T(k;s=i\omega+0) \tag{II.61}$$

III. THE PHOTON DISTRIBUTION FUNCTION IN REAL TIME

The solution of the Langevin equation (II.34) can be found by Laplace transform. Defining the Laplace transforms

$$\tilde{\mathcal{A}}_{\vec{k},l}(s) \equiv \int_0^\infty dte^{-st}\mathcal{A}_{\vec{k},l}(t) \;\; ; \;\; \tilde{\xi}_{\vec{k},l}(s) \equiv \int_0^\infty dte^{-st}\xi_{\vec{k},l}(t) \tag{III.1}$$

along with the Laplace transform of the photon polarization given by eqn. (II.60) we find the solution

$$\tilde{\mathcal{A}}_{\vec{k},l}(s) = \frac{\mathcal{E}^i_{\vec{k},l} + s\mathcal{A}^i_{\vec{k},l} + \tilde{\xi}_{\vec{k},l}(s)}{s^2+k^2+\tilde{\Sigma}_T(k;s)} \tag{III.2}$$

where

$$\mathcal{A}^i_{\vec{k},l} = \frac{1}{\sqrt{\Omega}}\int d^3x e^{i\vec{k}\cdot\vec{x}}\mathcal{A}^i_l(\vec{x}) \;\; ; \;\; \mathcal{E}^i_{\vec{k},l} = \frac{1}{\sqrt{\Omega}}\int d^3x e^{i\vec{k}\cdot\vec{x}}\mathcal{E}^i_l(\vec{x}) \tag{III.3}$$

and the superscript i refers to the initial conditions (II.33). The solution in real time can be written in a more compact manner as follows. Introduce the fundamental solution $f_k(t)$ of the equation of motion

$$\ddot{f}_k(t) + k^2 f_k(t) + \int_0^t dt' \; \Sigma_T(k;t-t')f_k(t') = 0 \tag{III.4}$$

obeying the initial conditions[1]

$$f_k(t=0) = 0; \quad \dot{f}_k(t=0) = 1 \tag{III.5}$$

Its Laplace transform is given by

$$\tilde{f}_k(s) = \frac{1}{s^2+k^2+\tilde{\Sigma}_T(k;s)} \tag{III.6}$$

which is recognized as the Laplace transform of the full transverse photon propagator.

The fundamental solution $f_k(t)$ is found by the inverse Laplace transform

$$f_k(t) = \int_C \frac{ds}{2\pi i}\frac{e^{st}}{s^2+k^2+\tilde{\Sigma}_T(k;s)} \tag{III.7}$$

---

[1] The lower limit in the integral $t=0$ simply reflects the choice of the initial time. If an arbitrary initial time is chosen t_0, the lower limit becomes t_0, since the $\Sigma_T(k;t-t')$ is manifestly time translational invariant, the solution of the equation of motion is a function of $t-t_0$.

where C stands for the Bromwich contour, parallel to the imaginary axis in the complex s plane to the right of all the singularities of $\tilde{f}(s)$ and along the semicircle at infinity for $\mathrm{Re}\, s < 0$. The singularities of $\tilde{f}(s)$ in the physical sheet are isolated single particle poles and multiparticle cuts along the imaginary axis. Thus the contour runs parallel to the imaginary axis with a small positive real part with $s = i\omega + \epsilon\,;\, -\infty \leq \omega \leq \infty$ and wraps around returning parallel to the imaginary axis with $s = i\omega - \epsilon\,;\, \infty > \omega > -\infty$, with $\epsilon = 0^+$. From the spectral representations (II.52,II.60)) one finds that $\tilde{\Sigma}_T(k, s = i\omega \pm \epsilon) = \mathrm{Re}\tilde{\Sigma}^R_T(k,\omega) \pm \mathrm{Im}\tilde{\Sigma}^R_T(k,\omega)$ and using that $\mathrm{Im}\Sigma^R_T(k,\omega) = -\mathrm{Im}\tilde{\Sigma}^R_T(k,-\omega)$ we find

$$f_k(t) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\sin(\omega t)\ \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega) + 2\omega\epsilon\right]}{\left[\omega^2 - k^2 - \mathrm{Re}\tilde{\Sigma}^R_T(k;\omega)\right]^2 + \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega) + 2\omega\epsilon\right]^2} \tag{III.8}$$

We have kept the infinitesimal $2\omega\epsilon\,;\, \epsilon \to 0^+$ to highlight the possibility of isolated quasiparticle poles in the case of vanishing imaginary part of the polarization.

The initial condition $\dot{f}_k(t=0) = 1$ results in the following sum rule

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\omega \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega) + 2\omega\epsilon\right]}{\left[\omega^2 - \omega_k^2 - \mathrm{Re}\tilde{\Sigma}^R_T(k;\omega)\right]^2 + \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega) + 2\omega\epsilon\right]^2} = 1 \tag{III.9}$$

In terms of the fundamental solution $f_k(t)$ given above, the solution of the Langevin equation (II.34) in real time is given by

$$\mathcal{A}_{\vec{k},l}(t;\mathcal{A}_i;\mathcal{E}_i;\xi) = \mathcal{A}^i_{\vec{k},l}\ \dot{f}_k(t) + \mathcal{E}^i_{\vec{k},l}\ f_k(t) + \int_0^t f_k(t-t')\ \xi_{\vec{k},l}(t')dt' \tag{III.10}$$

where the superscript i refers to the initial conditions (II.33).

This solution in real time represents the resummation of the Dyson series.

A. Number operator:

We *define* the photon number operator to be given by

$$\sum_{\lambda} \hat{N}_{k,\lambda}(t) = \frac{1}{2k\,Z}\left\{\dot{\hat{\mathbf{A}}}_{T,\vec{k}}(t)\cdot\dot{\hat{\mathbf{A}}}_{T,-\vec{k}}(t) + k^2\ \hat{\mathbf{A}}_{T,\vec{k}}(t)\cdot\hat{\mathbf{A}}_{T,-\vec{k}}(t)\right\} - \mathcal{C}_k \tag{III.11}$$

where the index λ labels the two independent transverse polarization states and according to asymptotic theory of interacting fields, Z is identified with the wave function renormalization for *asymptotic states*, namely Z is the wave-function renormalization constant in the *vacuum*. The constant $\mathcal{C}_k$ will be adjusted so as to subtract the photon number in the vacuum state. In free field theory $Z = 1\,, \mathcal{C}_k = 1$. In asymptotic theory the field $\hat{\mathbf{A}}_{T,\vec{k}}$ creates a single particle state of momentum k with amplitude $\sqrt{Z}$ out of the vacuum state. The quantities $Z\,,\mathcal{C}_k$ account for renormalization aspects in the definition of the particle number in an interacting field theory. A detailed study in ref. [15] shows that once Z and $\mathcal{C}_k$ are fixed by the wave function renormalization constant and the normal ordering subtraction in the vacuum, these terms cancel the zero temperature (vacuum) contributions. Therefore this definition of the number operator does indeed describe the number of photons *in the medium* at time t.

Introducing the real-time correlation functions

$$\langle A^+_{T,j}(\vec{k};t)A^+_{T,l}(-\vec{k};t')\rangle = \mathcal{P}^{j,l}(\hat{\mathbf{k}})\left[g^>_k(t,t')\Theta(t-t') + g^<_k(t,t')\Theta(t'-t)\right] \tag{III.12}$$

$$\langle A^-_{T,j}(\vec{k};t)A^-_{T,l}(-\vec{k};t')\rangle = \mathcal{P}^{j,l}(\hat{\mathbf{k}})\left[g^>_k(t,t')\Theta(t'-t) + g^<_k(t,t')\Theta(t-t')\right] \tag{III.13}$$

$$\langle A^-_{T,j}(\vec{k};t)A^+_{T,l}(-\vec{k};t')\rangle = \mathcal{P}^{j,l}(\hat{\mathbf{k}})\ g^>_k(t,t') \tag{III.14}$$

$$\langle A^-_{T,l}(-\vec{k};t')A^+_{T,j}(\vec{k};t)\rangle = \mathcal{P}^{j,l}(\hat{\mathbf{k}})\ g^<_k(t,t') \tag{III.15}$$

where the superscripts $\pm$ refer to the forward $(+)$ and backwards $(-)$ time branches. The expectation value of this number operator is defined in eq. (II.43), and assuming $\overline{\mathcal{A}^a_{i,\vec{k}}} = 0; \overline{\mathcal{E}^b_{i,-\vec{k}}} = 0$ its expectation value in the time evolved

density matrix is found to be given by

$$(2\pi)^3 \frac{dN}{d^3k d^3x} \equiv \sum_\lambda \langle \hat{N}_{k,\lambda}(t)\rangle = \frac{1}{2k\,Z}\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'} + k^2\right)\left[g^>_k(t,t') + g^<_k(t,t')\right]|_{t=t'} - \mathcal{C}_k \tag{III.16}$$

In terms of the center of mass field $\vec{\mathcal{A}}$ introduced in eq. (II.22) it is straightforward to find that the correlation function in the bracket in eq. (III.11) is given by

$$\langle \mathcal{A}^a_{\vec{k}}(t)\mathcal{A}^b_{-\vec{k}}(t')\rangle = \frac{\mathcal{P}^{ab}(\hat{\mathbf{k}})}{2}\left[g^>_k(t,t') + g^<_k(t,t')\right] \tag{III.17}$$

and the occupation number can be written in terms of the center of mass Wigner variable as follows

$$\sum_\lambda \langle \hat{N}_{k,\lambda}(t)\rangle = \frac{1}{2kZ}\left[\langle \dot{\vec{\mathcal{A}}}_{\vec{k}}(t)\cdot \dot{\vec{\mathcal{A}}}_{-\vec{k}}(t)\rangle + k^2 \langle \vec{\mathcal{A}}_{\vec{k}}(t)\cdot \vec{\mathcal{A}}_{-\vec{k}}(t)\rangle\right] - \mathcal{C}_k \tag{III.18}$$

where the expectation values are obtained as in eq. (II.43) and $\vec{\mathcal{A}}_{\vec{k}}(t)$ is the solution of the Langevin equation given by eq. (III.10).

Assuming for simplicity that $\overline{\mathcal{A}^a_{i,\vec{k}}} = 0; \overline{\mathcal{E}^b_{i,-\vec{k}}} = 0$, the expectation value of the number operator eq. (III.11) as defined by eq. (II.43) is given by

$$\begin{aligned}(2\pi)^3 \frac{dN}{d^3k d^3x} \equiv \sum_\lambda \langle \hat{N}_{k,\lambda}(t)\rangle \; &= \; \frac{1}{k\;Z}\left\{\frac{1}{2k}\left[1 + 2\mathcal{N}_k\right]\left[\ddot{f}^2_k(t) + 2k^2\;\dot{f}^2_k(t) + k^4\;f^2_k(t)\right]\right.\\ &\left. + \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\tilde{\mathcal{K}}_T(k,\omega)\left[|\mathcal{F}_k(\omega,t)|^2 + k^2|\mathcal{H}_k(\omega,t)|^2\right]\right\} - \mathcal{C}_k\end{aligned} \tag{III.19}$$

where we have introduced the auxiliary functions

$$\mathcal{H}_k(\omega,t) = \int_0^t d\tau f_k(\tau)e^{-i\omega\tau} \tag{III.20}$$

$$\mathcal{F}_k(\omega,t) = \int_0^t d\tau \dot{f}_k(\tau)e^{-i\omega\tau} \tag{III.21}$$

and $\tilde{\mathcal{K}}_T(k,\omega)$ is given by the fluctuation-dissipation relation eq. (II.58) and $\mathcal{N}_k$ is the initial photon distribution function.

This is one of the main results of this study. The expression (III.19) is truly *non-perturbative* since the function $f_k(t)$ given by (III.8) describes the real time evolution after a *Dyson* resummation of the photon propagator in terms of the geometric series with the polarization Σ to lowest order in α_{em} but in principle to all orders in α_s.

Thus it is clear that the formulation in terms of the non-equilibrium effective action for the physical transverse gauge fields described above leads to a novel non-perturbative approach to study photon production from a thermal source directly in real time.

IV. LOWEST ORDER IN PERTURBATION THEORY:

The results obtained above are general and as such may be unfamiliar within the context of photon production from a QGP in LTE. In order to establish the relationship to the usual approach we now obtain the photon yield in strict perturbation theory, namely to lowest order in α_{em}. The Laplace transform (III.6) to lowest order in the perturbative expansion in α_{em} is given by

$$\tilde{f}_k(s) = \frac{1}{s^2+k^2} - \frac{\tilde{\Sigma}_T(k;s)}{(s^2+k^2)^2} + \mathcal{O}(\alpha^2_{em}) \tag{IV.1}$$

The fundamental solution can be readily obtained by inverting the Laplace transform using eq. (II.60) and is found to be

$$f_k(t) = \frac{\sin[kt]}{k} + \delta f_k(t) + \mathcal{O}(\alpha_{em}^2) \tag{IV.2}$$

with

$$\begin{aligned}\delta f_k(t) = \; & \frac{\sin[kt]}{k}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\left\{\frac{1}{2k^2(\omega-k)} - \frac{1}{2k(\omega-k)^2}\right\} - \frac{t\cos[kt]}{k}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)}{2k(\omega-k)} \\ & + \int_{-\infty}^{\infty}\frac{d\omega}{\pi}\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\frac{\sin[\omega t]}{(\omega^2-k^2)^2}\end{aligned} \tag{IV.3}$$

Inserting this perturbative solution in eq. (III.19) and keeping terms consistently up to $\mathcal{O}(\Sigma_T) \sim \mathcal{O}(\alpha_{em})$ and setting $\mathcal{N}_k = 0$ we find to lowest order in α_{em}

$$(2\pi)^3\frac{dN(t)}{d^3k d^3x} = \left[\frac{1}{Z} - \mathcal{C}_k\right] + \frac{1}{k}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}2\,\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\,n(\omega)\,\frac{1-\cos[(\omega-k)t]}{(\omega-k)^2} \tag{IV.4}$$

The $1/Z$ in the first term is a consequence of introducing the wave function renormalization in the definition of the photon number in equation (III.11), setting $Z = 1; \mathcal{C}_k = 1$ corresponding to the free field expression, the first term vanishes. The second term is *exactly* the one obtained in a perturbative expansion in *real time* in reference [15]. This equivalence can be inferred by noticing that the transverse part of the polarization $\Pi_T(k,\omega)$ as defined in eq. (IV.9) in reference [15] and $\tilde{\Sigma}^R_T(k;\omega)$ as defined by eq. (II.19) above imply that $\Pi_T(k,\omega) \equiv 2\tilde{\Sigma}^R_T(k;\omega)$. Of course the perturbative study of reference [15] defined the number of photons as in free field theory, corresponding to setting $Z = 1; \mathcal{C}_k = 1$. Thus we see that the perturbative evaluation of the expression (III.18) to lowest order in α_{em} is exactly the same as obtained in the real time study in [15].

Furthermore, as emphasized in ref.[15] the asymptotic long time limit takes the form

$$\frac{dN(t)}{d^3x d^3k} = \frac{1}{(2\pi)^3\,k}\left[\frac{2\,\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega=k)}{e^{\frac{k}{T}}-1}\,t + \int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\frac{2\,\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)}{e^{\frac{\omega}{T}}-1}\,\mathcal{P}\frac{1}{(\omega-k)^2} + \mathcal{O}\left(\frac{1}{t}\right)\right]. \tag{IV.5}$$

Therefore in the asymptotic long time limit and to lowest order in α_{em} the *photon production rate*

$$\frac{dN(t)}{d^4x d^3k} = \frac{1}{(2\pi)^3}\frac{2\,\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega=k)}{k(e^{\frac{k}{T}}-1)}. \tag{IV.6}$$

is *exactly* the S-matrix result[15]. In the case that $\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega=k) = 0$ as for example in the hard thermal loop approximation[27, 28, 29] a logarithmic dependence replaces the linear time growth in eq. (IV.5)[15, 16].

A. Photon production from kinetics:

Kinetic theory provides an alternative approach to photon production. Within a simple kinetic description the time evolution of the number of photons in a phase space cell is given by a gain minus loss (master) type kinetic equation, which for the distribution function of each polarization is given by

$$\frac{d\,n_{k,\lambda}(t)}{dt} = [1+n_{k,\lambda}(t)]\Gamma^>_k - n_{k,\lambda}(t)\Gamma^<_k \tag{IV.7}$$

with the photon number (assuming an isotropic distribution) per polarization

$$n_{k,\lambda}(t) = (2\pi)^3\frac{d^3N_\lambda}{d^3x d^3k} \tag{IV.8}$$

and $\Gamma^{>}_k$; $\Gamma^{<}_k$ are the forward and backward rates which are computed using Fermi's Golden rule which is equivalent to the S-matrix calculation of transition rates. Detailed balance entails

$$\Gamma^{>}_k = e^{-\beta k}\,\Gamma^{<}_k \tag{IV.9}$$

The solution of the kinetic equation (IV.7) with the initial condition $n_{k,\lambda}(0) = 0$ gives the following time evolution for the sum over the polarization

$$n_k(t) = 2n_{eq}(k)(1 - e^{-\gamma_k t}) \;\; ; \;\; \gamma_k = \Gamma^{<}_k - \Gamma^{>}_k \tag{IV.10}$$

with $n_{eq}(k)$ the equilibrium Bose-Einstein distribution function for photons.

This solution illuminates two important aspects: i) the thermalization time scale is $\tau_k = 1/\gamma_k$ and ii) for $t << \tau_k$

$$n_k(t) = 2n_{eq}(k)\,\gamma_k\, t + \mathcal{O}(\gamma^2_k\; t^2) \tag{IV.11}$$

the photon production rate is precisely determined by the expression for the photon number during time scales much shorter than that for thermalization, namely

$$\frac{d\; n_k(t)}{dt} = 2n_{eq}(k)\gamma_k \tag{IV.12}$$

The relaxation rate of the distribution function $\gamma_k = 2\Gamma_k$ with Γ_k relaxation (damping) rate of the single-particle excitation, in this case the photon damping rate, which is given by

$$\Gamma_k = \frac{\mathrm{Im}\tilde{\Sigma}_T(k, \omega = k)}{2k} \tag{IV.13}$$

where the factor k in the denominator refers to the free photon mass shell. Accounting for the two transverse polarization state, one obtains the rate of photon production

$$\frac{d\; n_k(t)}{dt} = \frac{2\,\mathrm{Im}\tilde{\Sigma}_T(k, \omega = k)}{k(e^{\frac{k}{T}} - 1)} \tag{IV.14}$$

which is the result obtained above in eq. (IV.6). This simple kinetic description clearly shows the main physical assumptions and ingredients that enter in the computation of the photon production rate.

The kinetic equation (IV.7) is obtained from a full quantum field theory description by defining the photon number operator as in the free field quantum theory of photons, thus the time evolution of this operator is solely determined by the interaction[24]. The expectation value of the number operator is obtained in a perturbative expansion using the real time Feynman rules with free field photon propagators that include the distribution function just as in the equilibrium case[24]. The main point of this discussion is that in the quantum field theory formulation leading to the simple kinetic equation (IV.7) the photons propagate as *free particles*, namely with the free photon dispersion relation. This ignores the fact that in the medium photons propagate as collective modes, *not* as single particle excitations. In order to include the collective effects in the medium, which are more relevant for photons with momenta smaller than or of the order of the temperature[27, 28, 29], a non-perturbative description is required.

B. Kinetic interpretation of the S-matrix result:

The analysis of the previous section leads to a kinetic interpretation of the S-matrix result. The S-matrix approach extracts the forward rate $\Gamma^{>}_k$ by computing the transition probability in the infinite time limit, assuming $n(k) = 0$ and *ignoring the build up of the photon population*, namely the terms with $n_{k,\lambda}$ in the kinetic equation (IV.7), thus leading to

$$\frac{dn_k}{dt} = 2\Gamma^{>}_k \tag{IV.15}$$

Using the detailed balance relation eq. (IV.9) and the relation $\gamma_k = 2\Gamma_k$ with Γ_k given by eq. (IV.13) one finds that eq. (IV.15) is equivalent to the result (IV.14).

Obviously the buildup of the population can only be neglected during a time scale $t \ll 1/\Gamma_k$, however the S-matrix approach manifestly takes the time to infinity extracting only the terms that grow linearly in time in this limit, and ignoring terms that grow slower and that can make important contributions during a finite time scale[15, 16]. In *assuming* that the S-matrix approach is therefore valid only during a finite interval of time before the photon population builds up highlights the inconsistency of neglecting time dependent contributions associated with the finite lifetime. While this point has been emphasized in refs.[15, 16] the analysis based on the kinetic equation makes it explicit.

A kinetic interpretation of the S-matrix result as gleaned from the full non-perturbative solution of the kinetic equation (IV.10) first assumes no initial population, that the rate is constant in time and that the population does not build up, leading to

$$\frac{dN_{SM}}{d^4x d^3k} = \left.\frac{dN_{kin}}{d^4x d^3k}\right|_{t=0} = 2\Gamma^{>}_{k} \tag{IV.16}$$

with the factor 2 accounting for the two polarization states.

V. NON-PERTURBATIVE ASPECTS I: BREIT-WIGNER (NARROW WIDTH) APPROXIMATION:

Having confirmed that the lowest order in the strict perturbative expansion in α_{em} of the full expression (III.19) coincides with the results previously obtained in the literature, we now proceed to study non-perturbative aspects. The first stage of our study is to make contact with the kinetic approach to photon production studied in section IV A above.

For weak couplings (α_{em} *and* α_s) when the width of the quasiparticle is much smaller than its energy, the photon spectral density

$$\rho_\gamma(k;\omega) = \frac{1}{\pi}\frac{\left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\right]}{\left[\omega^2 - k^2 - \mathrm{Re}\tilde{\Sigma}^R_T(k;\omega)\right]^2 + \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\right]^2} \tag{V.1}$$

features a pole in the second (unphysical) Riemann sheet but near the real axis at the position of the "quasiparticle" pole, which is a solution of the equation

$$\omega_p^2(k) - k^2 - \mathrm{Re}\tilde{\Sigma}^R_T(k;\omega_p(k)) = 0 \tag{V.2}$$

The imaginary part of the transverse photon self-energy evaluated at $\omega = \omega_p(k)$ determines the width or damping rate of the single quasiparticle excitation. If $\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega_p(k)) = 0$ then the quasiparticle is stable corresponding to a true pole in the physical sheet. If $\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega_p(k)) \neq 0$ the quasiparticle pole moves off the physical sheet becoming a resonance. In weak coupling the spectral density can be approximated by a Breit-Wigner form near the quasiparticle pole

$$\rho_\gamma(k;\omega \approx \omega_p(k)) = \frac{\mathcal{Z}_k}{2\pi\omega_p(k)}\;\frac{\Gamma_k}{(\omega-\omega_p(k))^2 + \Gamma_k^2} \tag{V.3}$$

with the residue at the quasiparticle pole and the width given by

$$\mathcal{Z}_k^{-1} = 1 - \left.\frac{\partial\,\mathrm{Re}\tilde{\Sigma}^R_T(k;\omega)}{\partial\omega^2}\right|_{\omega=\omega_p(k)} \quad ; \quad \Gamma_k = \mathcal{Z}_k\frac{\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega_p(k))}{2\omega_p(k)} \tag{V.4}$$

The residue at the quasiparticle pole $\mathcal{Z}_k$ is in principle different from Z the wave function renormalization that enters in the asymptotic theory definition of the photon number (III.11). The reason for the difference is that the

photons that are measured in the detector are asymptotic states, hence Z in this definition should refer to the *vacuum* value and not the in-medium residue at the quasiparticle pole.

In strict perturbation theory the connection with the kinetic approach which applies to the distribution function of photons of dispersion relation $\omega_p(k) = k$ is established by taking $Z = 1; \mathcal{Z}_k = 1\,; \omega_p(k) = k\;, \mathcal{C}_k = 1$, assuming that there are no initial photons present, namely $\mathcal{N}_k = 0$, and assuming the Breit-Wigner form of the spectral density (V.3) in the full range of ω, not just near the quasiparticle pole. Under these assumptions

$$\rho_\gamma(k;\omega) = \frac{1}{2\pi k}\,\frac{\Gamma_k}{(\omega-k)^2+\Gamma_k^2} \quad ; \quad \Gamma_k = \frac{\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega=k)}{2k} \tag{V.5}$$

In the narrow width approximation $\Gamma_k \ll k$ the fundamental solution is given by

$$f_k(t) = \frac{\sin[kt]}{k} e^{-\Gamma_k t} \tag{V.6}$$

Inserting this solution and neglecting terms of $\mathcal{O}(\Gamma_k^2)$ in the numerator in eq. (III.19), and using the fluctuation-dissipation relation (II.58) we find

$$(2\pi)^3 \frac{dN(t)}{d^3k d^3x} = e^{-2\Gamma_k t} - 1 + \frac{1}{2k}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\,\frac{[1+2n(\omega)]\,\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)}{(\omega-k)^2+\Gamma_k^2}\left[1+e^{-2\Gamma_k t}-2e^{-\Gamma_k t}\cos[(\omega-k)t]\right] \tag{V.7}$$

In the weak coupling limit, assuming a narrow Breit-Wigner spectral density, namely taking $\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega) \sim 2k\,\Gamma_k$ and using that the spectral density is sharply peaked at $\omega \sim k$ we can replace $n(\omega) \equiv n_{eq}(k)$ with $n_{eq}(k)$ the Bose-Einstein distribution function for photons. The integrals can be done straightforwardly and we find

$$(2\pi)^3 \frac{dN(t)}{d^3k d^3x} = 2n_{eq}(k)(1-e^{-2\Gamma_k t}) \tag{V.8}$$

This result is the same as the solution of the kinetic equation (IV.10) since $\gamma_k = 2\Gamma_k$. The assumptions leading to eq. (V.8), namely weak coupling, neglecting self-energy corrections to the dispersion relation as well as renormalization effects (wave-function renormalization) and the narrow-width-Breit-Wigner approximation for the spectral density are all approximations invoked in the Fermi's golden rule approach to the kinetic description.

We note that even under all these assumptions, the result eq. (V.8) is truly *non-perturbative* and a result of the non-perturbative Dyson-like resummation of the photon propagator manifest in the function $f_k(t)$. The strict perturbative evaluation as presented in the previous section reveals that when $\mathrm{Im}\tilde{\Sigma}(k,\omega\sim k)\neq 0$, the photon yield grows linearly in time, a consequence of Fermi's golden rule (or secular terms). Such strict perturbative evaluation clearly is restricted to a time interval $t \ll \Gamma_k^{-1}$ since for $t \gg \Gamma_k^{-1}$ the photon yield attains the equilibrium form given by the Bose-Einstein distribution function. The secular terms (terms that grow in time) that appear in the strict perturbative expansion of eq. (V.8) are typically manifest as pinch singularities in finite temperature field theory[30]. A dynamical renormalization group program has been recently introduced that provides a resummation of these secular terms and leads to their exponentiation [31]. Thus, making contact with the dynamical renormalization group resummation of terms that grow in time, it is clear that the formulation presented above leads to a resummation of the perturbative series. The description of photon production given by our result eq. (III.19) not only coincides with the perturbative result in the strict perturbative expansion, but provides a systematic Dyson-like resummation of the perturbative series leading to a uniform expansion in the coupling valid at all times. This resummation is similar to that implied by the dynamical renormalization group approach of ref.[31] and reveals the physics of the relaxation of the population towards equilibrium as well as the corresponding time scales.

VI. NON-PERTURBATIVE ASPECTS II: PHOTONS OR PLASMONS?

The discussion in the previous two sections above confirmed that the real time description of photon production describes both the lowest order results known previously and is capable of describing the thermalization of photons.

We now study non-perturbative aspects of the production and *propagation* of photons in a locally equilibrated QGP.

An important result from the hard thermal loop program in finite temperature gauge field theories[27, 28, 29] is that even for weak coupling the perturbative expansion must be resummed for momenta $k \leq \omega_{pl}$ with ω_{pl} the plasmon

mass. To lowest order in the HTL program for $N_c = 3$ and two (massless, up and down) quarks, the plasmon mass is given by

$$\omega_{pl} = \sqrt{\frac{5}{27}}\; eT = 13.033 \left(\frac{T}{100\ \text{Mev}}\right)\ \text{Mev} \tag{VI.1}$$

which for example for the temperature expected at RHIC $T \sim 300$ Mev $\omega_{pl} \sim 40$ Mev.

The real and imaginary parts of the transverse photon polarization in the HTL approximation are given by

$$\text{Re}\tilde{\Sigma}^R_T(k;\omega) = k^2(x^2-1) - \frac{3\omega^2_{pl}}{2}\left[x^2 + \frac{1}{2}x(1-x^2)\ln\left|\frac{x+1}{x-1}\right|\right] \quad ; \quad x \equiv \frac{\omega}{k} \tag{VI.2}$$

$$\text{Im}\tilde{\Sigma}^R_T(k;\omega) = \frac{3\pi}{4}\omega^2_{pl}\ \frac{\omega}{k}\left(1-\frac{\omega^2}{k^2}\right)\Theta(k^2-\omega^2) \tag{VI.3}$$

In the HTL approximation, the position of the plasmon pole as a function of momentum k is determined by the solution(s) of the trascendental equation

$$x_p^2 - 1 - \frac{3\omega^2_{pl}}{2k^2}\left[x_p^2 + \frac{1}{2}x_p(1-x_p^2)\ln\left[\frac{x_p+1}{x_p-1}\right]\right] = 0 \quad ; \quad x_p = \frac{\omega_p(k)}{k} \tag{VI.4}$$

The numerical solution for the dispersion relation is shown in fig. (1) for $T \sim 300$ Mev corresponding to a (transverse) plasma frequency $\omega_{pl} = 39.099$ Mev.

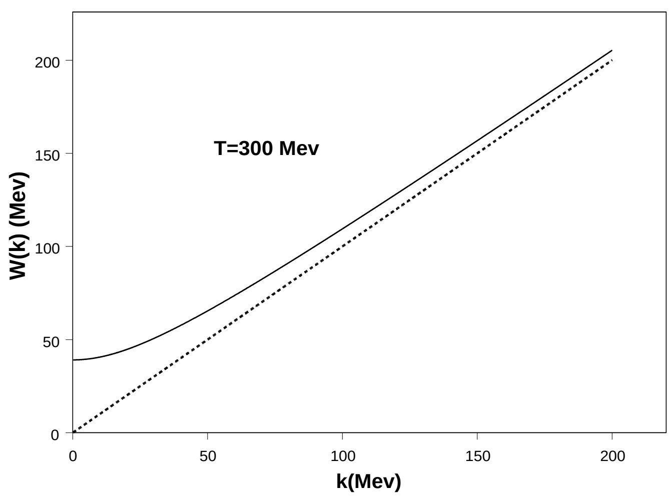

FIG. 1: The dispersion relation $\omega_p(k)$ (in Mev) vs. k (in Mev/c) in the HTL approximation for $\omega_{pl} = 39.099$ Mev, corresponding to a temperature $T = 300$ Mev . The solid line is the plasmon and the dashed line the free photon dispersion relations respectively.

Even for momenta $k \sim 200$ Mev $\gg \omega_{pl}$ the difference between the plasmon and the free particle dispersion relation is about 15%. The real time solution $f_k(t)$ in the HTL approximation is given by

$$\begin{aligned} f_k(t) &= \mathcal{Z}_k \frac{\sin[\omega_p(k)\,t]}{\omega_p(k)} + f_c(k,t) \\ f_c(k,t) &= \int_{-k}^{k}\frac{d\omega}{\pi}\frac{\sin(\omega t)\ \text{Im}\tilde{\Sigma}^R_T(k;\omega)}{\left[\omega^2-k^2-\text{Re}\tilde{\Sigma}^R_T(k;\omega)\right]^2 + \left[\text{Im}\tilde{\Sigma}^R_T(k;\omega)\right]^2} \end{aligned} \tag{VI.5}$$

where $\mathcal{Z}_k$ is the residue at the plasmon pole (wavefunction renormalization) given by

$$\mathcal{Z}_k = \left[1 - \frac{1}{2\omega_p(k)} \frac{\partial \mathrm{Re}\tilde{\Sigma}_T(k;\omega)}{\partial \omega}\Bigg|_{\omega=\omega_p(k)}\right]^{-1} \tag{VI.6}$$

The sum rule (III.9) entails the following

$$\mathcal{Z}_k + \int_{-k}^{k} \frac{d\omega}{\pi} \frac{\omega \ \mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)}{\left[\omega^2 - k^2 - \mathrm{Re}\tilde{\Sigma}^R_T(k;\omega)\right]^2 + \left[\mathrm{Im}\tilde{\Sigma}^R_T(k;\omega)\right]^2} = 1 \tag{VI.7}$$

The residue at the plasmon pole $\mathcal{Z}_k$ is shown as a function of k for $T \sim 300$ Mev in figure (2) below.

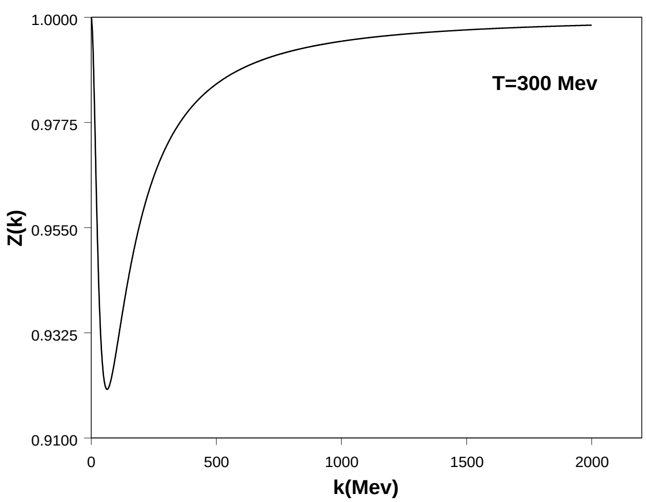

FIG. 2: The residue at the plasmon (quasiparticle) pole for $\omega_{pl} = 39.099$ Mev, corresponding to a temperature $T = 300$ Mev .

Figure (3) below displays the continuum contribution $f_c(k,t)$ given by the integral expression in eq. (VI.5) for the values $k = 50; 100; 150$ Mev/c respectively for $T \sim 300$ Mev. It is clear from this figure that the continuum contribution is perturbatively small, of $\mathcal{O}(\alpha_{em})$ and damps out on time scales of ~ 10 fm/c or longer even for large momenta. This time scale is of the order of the lifetime of the QGP expected at RHIC or LHC, hence $f_c(k,t)$ will contribute to $\mathcal{O}(\alpha_{em})$ during the lifetime of the QGP.

The main point of this discussion is to highlight that once the photon is produced at a space-time vertex, it propagates in the medium as a *plasmon* collective mode, not as a free electromagnetic plane wave. The propagation of the photon in the medium as a collective mode has obviously nothing to do with the thermalization of the produced photon and only depends on the wavelength of the photon being smaller than the spatial size of the system. The situation is similar to an electromagnetic wave propagating in a dispersive medium with an index of refraction, the mean free path of the photon in the medium can be larger than the spatial extension of the medium itself, but the electromagnetic wave propagates (almost undamped) as a combination of normal modes in the medium. This point has been advanced by Weldon[32] within the context of dilepton production in a thermalized QGP[2].

This point can be simply illustrated by considering the average over the noise of the solution (III.10) with an initial value of the expectation value of the transverse gauge field and its time derivative corresponding to the case of a

[2] D.B. thanks Abhijit Majumder for discussions on this point

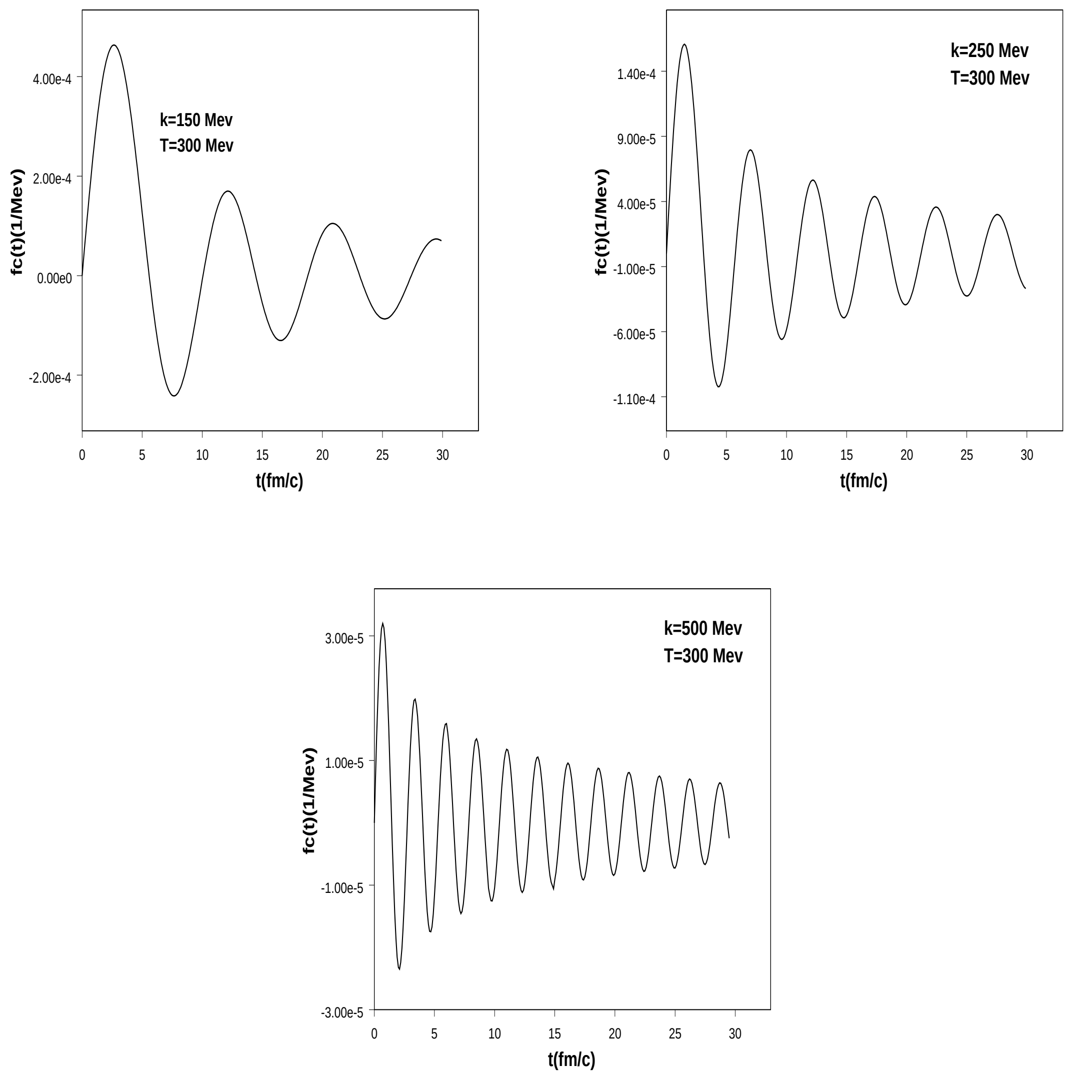

FIG. 3: The continuum contribution $f_c(k,t)$ given by the integral in (VI.5) for $k = 150; 250; 500$ Mev respectively as a function of t, for $T \sim 300$ Mev .

positive energy plane wave, namely

$$\mathcal{A}^i_{\vec{k},l} = \frac{\vec{\epsilon}_{\vec{k},l}(\lambda)}{\sqrt{2k}} \quad ; \quad \mathcal{E}^i_{\vec{k},l} = -ik\,\mathcal{A}^i_{\vec{k},l} \tag{VI.8}$$

with $\vec{\epsilon}_{\vec{k},l}(\lambda)$ being a transverse polarization vector.

The propagation of this initial state is given by eq. (III.10). The noise average eq. (II.38) implies

$$\langle \mathcal{A}_{\vec{k},l}(t)\rangle = \vec{\epsilon}_{\vec{k},l}(\lambda)\left[e^{i\omega_p(k)t}\;\frac{\mathcal{Z}_k}{2\sqrt{2k}}\left(1-\frac{k}{\omega_p(k)}\right)+e^{-i\omega_p(k)t}\;\frac{\mathcal{Z}_k}{2\sqrt{2k}}\left(1+\frac{k}{\omega_p(k)}\right)+\frac{1}{\sqrt{2k}}\dot{f}_c(k,t)-i\sqrt{\frac{k}{2}}f_c(k,t)\right] \tag{VI.9}$$

Clearly the propagation of the transverse electromagnetic wave is not free. While the continuum contribution $f_c(k,t)$ vanishes at long time, its presence guarantees that the initial conditions (VI.8) are fulfilled via the sum rule (VI.7).

A. Beyond HTL

The real and imaginary parts of the (transverse) photon self energy given by eqns. (VI.2), (VI.3) are only valid in the hard thermal loop approximation $k, \omega \ll T$. The full one loop expression for the finite temperature contribution to the imaginary part of the transverse photon self energy is given by

$$\mathrm{Im}\tilde{\Sigma}^{(1)}_{T}(k;\omega) = \mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega) + \mathrm{Im}\tilde{\Sigma}^{(1)}_{2P}(k;\omega) \tag{VI.10}$$

where $\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)$; $\mathrm{Im}\tilde{\Sigma}^{(1)}_{2P}(k;\omega)$ are the Landau damping and two particle cut respectively, given by[15]

$$\begin{aligned}
\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega) &= \frac{5}{3}\,\alpha_{em}\,T^2\left(1-\frac{\omega^2}{k^2}\right)\Bigg[\frac{k}{T}\ln\left(\frac{1+e^{-W_-}}{1+e^{-W_+}}\right) + \frac{2T}{k}\sum_{m=1}^{\infty}(-1)^{m+1}\left(\frac{2}{m^3}+\frac{k}{T\,m^2}\right)\times \\
&\qquad \left(e^{-m\,W_-}-e^{-m\,W_+}\right)\Bigg]\Theta(k^2-\omega^2)\,\mathrm{sign}(\omega) && \text{(VI.11)}\\
\mathrm{Im}\tilde{\Sigma}^{(1)}_{2P}(k;\omega) &= \frac{5}{3}\,\alpha_{em}\,T^2\left(\frac{\omega^2}{k^2}-1\right)\Bigg[\frac{k}{T}\ln\left(\frac{1+e^{-W_+}}{1+e^{-W_-}}\right) - \frac{2T}{k}\sum_{m=1}^{\infty}(-1)^{m+1}\Big[\frac{2}{m^3}\left(e^{-m\,W_-}-e^{-m\,W_+}\right) \\
&\qquad -\frac{k}{T\,m^2}\left(e^{-m\,W_-}+e^{-m\,W_+}\right)\Bigg]\Theta(\omega^2-k^2)\,\mathrm{sign}(\omega)\;, && \text{(VI.12)}\\
W_{\pm} &= \left|\frac{|\omega|\pm k}{2T}\right|
\end{aligned}$$

and the superscript (1) refers to one-loop.

The real part of the self energy is found by the dispersion relation eq. (II.51). The two particle cut contribution above the light cone would lead to an imaginary part for the plasmon since the plasmon dispersion relation lies above the light cone. This imaginary part is obviously related to the decay of the plasmon into *bare massless* fermion-antifermion pairs. Clearly this is unphysical, finite temperature self-energy corrections to the fermion lines lead to a thermal mass for the intermediate fermions which in the HTL approximation is at least 22% larger than the plasmon mass[3], thus preventing plasmon decay into fermion quasiparticles. The correct imaginary part of the transverse photon self energy above the light cone requires a non-perturbative resummation that involves not only fermion self-energy corrections but also vertex corrections to satisfy the Ward identities. While the damping rate for a non-abelian plasmon at rest has been computed in ref.[33] and that for the abelian counterpart was estimated for large k in ([32]) the calculation of the imaginary part of the (transverse) photon self energy for all values of ω, k is not yet available. What *is* available in the literature is the photon production rate calculated by the S-matrix approach to lowest order in α_{em} and up to leading logarithmic order in α_s[12]. From this result we can extract $Im\tilde{\Sigma}^{R}_{T}(\omega = k, k)$ by making use of the relation eq. (IV.6).

The analysis presented above indicates that in order to obtain the complete real time dependence of $f_k(t)$, hence the real time photon yield, what is needed is the photon (transverse) self energy for *all values* of $\omega; k$. Below the light cone, the Landau damping contribution (VI.11) is the leading order result, of $\mathcal{O}(\alpha_{em}\alpha_s^0)$ whereas above the light cone a resummation of the fermion lines and vertex is required for a consistent analysis of the plasmon width. Since the full imaginary part of the transverse photon self-energy above the light cone is not available, only its value on the *free photon* mass shell, namely $\omega = k$, we *model* the imaginary part of the photon self energy in the full range of frequency as

$$\mathrm{Im}\tilde{\Sigma}(k;\omega) \approx \mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega) + \mathrm{Im}\tilde{\Sigma}^{PP}(k;\omega = k)\,\Theta(\omega^2-k^2)\,\mathrm{sign}(\omega) \tag{VI.13}$$

where $\mathrm{Im}\tilde{\Sigma}^{PP}(k;\omega = k)$ is extracted from the S-matrix photon production rate eq. (IV.6) and the leading logarithmic result quoted in ref.[12], resulting in the following expression for the imaginary part on the *free photon mass shell*

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[3] Even much larger than this estimate when the value for α_s given by eq. (VI.15) is included through gluon contributions to the fermion self energy.

$$\text{Im}\tilde{\Sigma}^{PP}(k;\omega=k) = \frac{20\pi T^2}{9}\ \alpha_{em}\ \alpha_s(T)\ \tanh\left[\frac{k}{2T}\right]\left[\ln\left(\frac{\sqrt{3}}{4\pi\alpha_s(T)}\right) + C_{tot}\left(\frac{k}{T}\right)\right]$$
$$C_{tot}(z) = \frac{1}{2}\ln(2z) + \frac{0.041}{z} - 0.3615 + 1.01\ e^{-1.35\,z} + \sqrt{\frac{4}{3}}\left[\frac{0.548}{z^{\frac{3}{2}}}\ \ln\left(12.28+\frac{1}{z}\right) + \frac{0.133\ z}{\sqrt{1+\frac{z}{16.27}}}\right] \quad \text{(VI.14)}$$

The strong coupling α_s is a function of temperature, we will use the lattice parametrization [34] for the temperature dependence of the strong coupling $\alpha_s(T)$ given by

$$\alpha_s(T) = \frac{6\pi}{29\ln\frac{8T}{T_c}} \quad ; \quad T_c \sim 0.16 \text{ Gev} . \quad \text{(VI.15)}$$

Although this lattice fit is valid at high temperatures and certainly not near the hadronization phase transition, we will assume its validity in the temperature range relevant for RHIC in order to obtain a numerical estimate of the photon yield. We note, however, that at the temperature expected at RHIC or LHC $T \sim 300-400$ Mev $\alpha_s(0.3\text{ Gev}) \sim 0.24$ and the validity of the perturbative expansion in α_s (even with leading logarithmic corrections) is at best questionable.

The assumption that the imaginary part is constant above the light cone is consistent with the Breit-Wigner approximation. The one-loop Landau damping contribution below the light cone leads to the plasmon dispersion relation and is the leading order ($\mathcal{O}(\alpha_{em})$) contribution to the photon self-energy. The damping of the plasmon excitation is *not* correctly described by this Breit-Wigner form, since the correct plasmon damping rate is given by (see eq. (V.4))

$$\Gamma_k = \frac{\mathcal{Z}_k}{2\omega_p(k)}\ \text{Im}\tilde{\Sigma}_T[k,\omega=\omega_p(k)] \quad \text{(VI.16)}$$

However, since $\omega_p(k)$ differs from k by less than 10% for the relevant range $k \geq 200$ Mev, and *assuming* that $\text{Im}\tilde{\Sigma}_T(k,\omega)$ is a smooth function of ω near $\omega_p(k)$ it is reasonable to assume that $\text{Im}\tilde{\Sigma}_T(k,\omega=\omega_p(k)) \sim \text{Im}\tilde{\Sigma}_T(k,\omega=k)$. A reliable estimate of the error incurred with this approximation requires knowledge of $\text{Im}\tilde{\Sigma}_T(k,\omega)$ for all values of $\omega \geq k$ near the plasmon pole which is not yet available and its calculation is certainly beyond the scope of this article.

Consistent with the above approximation for the imaginary part, the lowest order $\mathcal{O}(\alpha_{em})$ contribution for the real part is obtained by the dispersion relation eq. (II.51). In order to guarantee the vanishing of the magnetic mass in the abelian theory[28], we subtract the dispersion relation at zero frequency. The real part is therefore given by

$$\text{Re}\tilde{\Sigma}^{(1)}_T(k,k_0) = -\frac{2k_0^2}{\pi}\int_0^k d\omega\ \mathcal{P}\frac{\text{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{\omega(\omega^2-k_0^2)} \quad \text{(VI.17)}$$

where we have performed a subtraction at $k_0 = 0$ to ensure the vanishing of the magnetic mass and used the property (II.53), $\mathcal{P}$ stands for the principal part.

The spectral density eq. (V.1) with the real and imaginary parts of the self energy given by eq.(VI.17) and (VI.13), (VI.11),(VI.14) respectively features a sharp peak above the lightcone at a value of the frequency given by

$$\omega_p^2(k) - k^2 - \text{Re}\tilde{\Sigma}^{(1)}_T(k;k_0=\omega_p(k)) = 0 \quad \text{(VI.18)}$$

We find numerically that the plasmon dispersion relation $\omega_p(k)$ and residue at the plasmon pole $\mathcal{Z}_k$ are remarkably similar to that obtained in the HTL approximation for momenta $k \geq 200$ Mev for $300 \leq T \leq 500$ Mev displayed in figures (1,2) respectively. This similarity was already pointed out in ref.[35] and our numerical study confirms the results obtained in that reference. This fact can be understood easily since for $k \ll T$ the full one loop imaginary part reduces to that in the HTL approximation, and for $k \gg T$ the corrections are truly perturbative. Above the light cone the photon spectral density can be very well approximated by the Breit-Wigner form

$$\rho_\gamma(k;\omega>k) = \frac{\mathcal{Z}_k}{2\pi\omega_p(k)}\ \frac{\Gamma_k}{(\omega-\omega_p(k))^2+\Gamma_k^2} \quad \text{(VI.19)}$$

with the residue at the plasmon pole and the width given by

$$\mathcal{Z}_k^{-1} = 1 - \frac{\partial\,\mathrm{Re}\tilde{\Sigma}^{(1)}_T(k;\omega)}{\partial\omega^2}\Bigg|_{\omega=\omega_p(k)} \quad ; \quad \Gamma_k = \mathcal{Z}_k \frac{\mathrm{Im}\tilde{\Sigma}^{PP}(k;\omega=k)}{2\omega_p(k)} \tag{VI.20}$$

For $300 \leq T \leq 450$ Mev $\omega_p(k); \mathcal{Z}_k$ for $k \geq 200$ Mev computed from equations (VI.13), (VI.17) and (VI.20) are numerically indistinguishable from those shown in figures (1,2) respectively, while Γ_k is displayed in fig. (4).

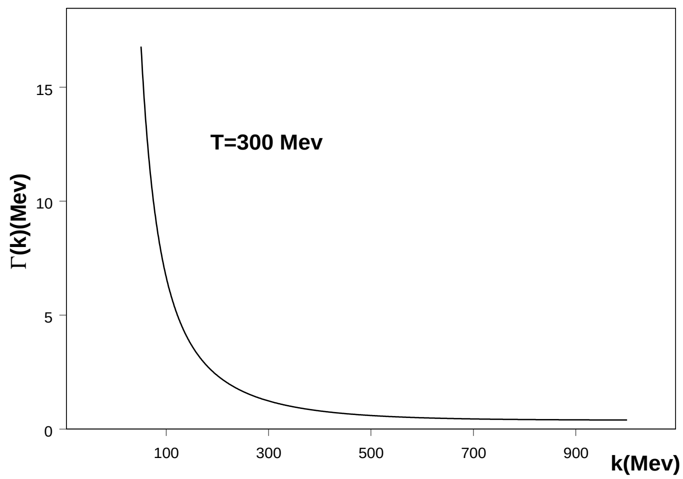

FIG. 4: The approximation to the plasmon width given by (VI.20) for $\omega_{pl} = 39.099$ Mev, corresponding to a temperature $T = 300$ Mev .

This figure in combination with the kinetic analysis of sections (IV A) reveal an important aspect: the solution of the kinetic equation (IV.10) shows that the time scale for thermalization of a photon with momentum k is $t_{th;k} = 1/2\Gamma_k$. Figure (4) shows that for photons with $k \leq 200$ Mev the time scale for thermalization is $\lesssim 15$ fm/c which is of the same order as the lifetime of the QGP expected at RHIC and LHC. Hence low energy photons are expected to thermalize as *quasiparticles* in a QGP in LTE. This an important aspect that is completely missed by the S-matrix approach which would assign these photons a constant rate of emission even when they thermalize with the medium during the lifetime of the QGP.

Below the light cone, the spectral density is determined by the one loop Landau damping contribution and is given by eq. (V.1) with the real part of the photon self-energy given by eq. (VI.17) and the imaginary part given by $\tilde{\Sigma}^{(1)}_{LD}(k;\omega)$ given by eq. (VI.11).

In this approximation, the real time solution $f_k(t)$ is given by

$$\begin{aligned} f_k(t) &= \mathcal{Z}_k \frac{\sin[\omega_p(k)\,t]}{\omega_p(k)}\; e^{-\Gamma_k\,t} + f_c(k,t) \\ f_c(k,t) &= \int_{-k}^{k} \frac{d\omega}{\pi} \frac{\sin(\omega t)\;\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{\left[\omega^2 - k^2 - \mathrm{Re}\tilde{\Sigma}^{(1)}_T(k;\omega)\right]^2 + \left[\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)\right]^2} \end{aligned} \tag{VI.21}$$

This solution represents a *Dyson* resummation of the lowest order contributions to the self-energy above and below the lightcone within the approximations detailed above. The initial condition $\dot{f}_c(k,0) = 1$ is satisfied by the sum rule (VI.7) which we confirmed numerically. Thus the contribution from the Landau damping cut is necessary to fulfill the initial condition and the sum rule.

With this real time solution, we now proceed to obtain the number of photons emitted from a QGP in LTE up to a time t by inserting this solution eq. (VI.21) in the expressions (III.19-III.21). Since we are only computing the finite

temperature contributions to the photon self-energy and therefore neglecting the vacuum terms, we consistently set $Z = 1$ in eq. (III.19). As discussed in detail in ref.[15] the zero temperature contribution to the self-energy yields the number of photons in the vacuum cloud which is precisely cancelled by the *vacuum* wave function renormalization. Consistent with this result in ref.[15] we neglect the zero temperature contribution to the self-energy and set $Z = 1$, since both contributions cancel each other out in the photon number[15]. Thus we focus solely on the photons created in the medium. We have obtained the expression (III.19) under the assumption that the initial density matrix is factorized and gaussian for photons. Of course this assumption can be relaxed to contemplate initial states with correlations between quarks and photons, however as discussed in detail in ref.[15] such density matrix will *not* commute with H_{QCD} since any entanglement between quark and photon states will involve the quark current. Furthermore any correlated initial state will necessarily have to be modelled to include the initial preparation during and after the collision as discussed in detail in ref.[15]. In what follows we will study the simplest assumption, namely that the initial state is uncorrelated and with no photons. Accordingly, we set $\mathcal{N}_k = 0$ in eq. (III.19). Introducing the solution (VI.21) into eq. (III.19) leads to many different contributions. We will explicitly keep terms that are of lowest order in α_{em} in the *uniform* expansion in the electromagnetic coupling. We emphasize a *uniform* expansion to contrast with a *naive* expansion which features terms that grow in time, namely secular terms. In the many terms obtained we only keep those whose numerators are manifestly of $\mathcal{O}(\alpha_{em})$ and neglect those which are of higher orders. Specifically we find,

$$\frac{dN}{d^3x d^3k} = \frac{1}{(2\pi)^3} \sum_{i=1}^{4} F_i(k,T,t) \tag{VI.22}$$

with the following contributions

$$\begin{aligned}
F_1(k,T,t) &= 2n(\omega_p(k))\mathcal{Z}_k(1-e^{-2\Gamma_k t}) && \text{(VI.23)}\\
F_2(k,T,t) &= 2\frac{e^{-2\Gamma_k t}}{k}\int_{-k}^{k}\frac{d\omega}{\pi}\frac{\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{[\omega-\omega_p(k)]^2+\Gamma_k^2}\left[1-\cos((\omega-\omega_p(k))t)\right] && \text{(VI.24)}\\
F_3(k,T,t) &= (\mathcal{Z}_k-1)(1-e^{-\Gamma_k t})^2 && \text{(VI.25)}\\
F_4(k,T,t) &= \frac{(1-e^{-\Gamma_k t})^2}{2k}\int_{-k}^{k}\frac{d\omega}{\pi}\frac{\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{[\omega-\omega_p(k)]^2+\Gamma_k^2}(1+2n(\omega)) && \text{(VI.26)}
\end{aligned}$$

with the wave function renormalization given to lowest order by

$$\mathcal{Z}_k = 1 - \int_{-k}^{k}\frac{d\omega}{\pi}\frac{\omega\ \mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{\left[\omega^2-k^2-\mathrm{Re}\tilde{\Sigma}^{(1)}_{T}(k;\omega)\right]^2+\left[\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)\right]^2} \tag{VI.27}$$

This expression summarizes the main result of this article: a non-perturbative expression the determines the direct photon yield directly in real time. It provides a Dyson-like resummation of the naive perturbative expansion, it is uniformly valid at all times and determines the yield to lowest order in α_{em}.

A strict perturbative expansion of this result up to $\mathcal{O}(\alpha_{em})$ leads to the following yield

$$\frac{dN_{PT}}{d^3x d^3k} = \frac{1}{(2\pi)^3}\left[2n(k)2\Gamma_k\ t + \frac{2}{k}\int_{-k}^{k}\frac{d\omega}{\pi}\frac{\mathrm{Im}\tilde{\Sigma}^{(1)}_{LD}(k;\omega)}{[\omega-k]^2}\left[1-\cos((\omega-k)t)\right]\right] \tag{VI.28}$$

Taking the asymptotic long time limit in this expression, the second term grows $\propto \ln(t)$ at long times[15] hence it would be subleading and therefore neglected, while the first term yields the S-matrix result for the rate

$$\frac{dN_{SM}}{d^4x d^3k} = 2n(k)(2\Gamma_k) = \frac{2\ \mathrm{Im}\tilde{\Sigma}^{PP}(k;\omega=k)}{(2\pi)^3\ k(e^{\frac{k}{T}}-1)} \tag{VI.29}$$

with $\mathrm{Im}\tilde{\Sigma}^{PP}(k;\omega=k)$ given by eq. (VI.14).

Thus it is clear that our final result for the photon yield during a finite time t given by eqns. (VI.22-VI.26) is a truly non-perturbative resummation of the naive perturbative expansion that incorporates real-time processes not captured by the S-matrix approach.

Furthermore this result clearly reveals several physical aspects that are missed by the S-matrix approach and that underlie its shortcomings:

- In the asymptotic long time limit, the only terms that survive are (VI.23,VI.25,VI.26) which in the perturbative limit $\mathcal{Z}_k = 1 + \mathcal{O}(\alpha_{em})$, reveal the correct physics: in a QGP in equilibrium with infinite lifetime, photons will thermalize as *quasiparticles* in the medium. Their abundance is determined by the Bose-Einstein distribution function for the correct quasiparticle frequency, and the wave function renormalization measures the weight of the quasiparticle in the spectral density.

- The width Γ_k is formally of $\mathcal{O}(\alpha_{em})$, however expanding the expressions (VI.23-VI.26) in Γ_k, namely $e^{-\Gamma_k t} \sim 1 - \Gamma_k t + \frac{1}{2}\Gamma_k^2 t^2 + \mathcal{O}(\Gamma_k^3 t^3)$ will lead to *secular* terms in time, namely terms that grow in time *to all orders in perturbation theory*. Similarly, since $\omega_p(k) - k$ is formally of $\mathcal{O}(\alpha_{em})$ one would be tempted to expand the argument of the cosine in eq. (VI.24), this again will lead to secular terms in time. Finally a naive perturbative expansion will attempt to replace $\omega_p(k) \to k, \Gamma_k \to 0$ in the denominators of eq. (VI.24) and (VI.26), however this will lead to logarithmic divergences at the Landau damping threshold which manifest themselves as logarithmic secular terms in time[15]. The S-matrix approach computes the transition probability per unit space-time volume by taking the infinite time limit and extracts the linear term in time which gives a constant rate. Obviously taking the long time limit is manifestly ignoring the fact that however long the photon relaxation time, eventually photons will thermalize in the long time limit. A *kinetic* interpretation of the S-matrix result, as discussed in section (IV A) is given by

$$\frac{dN_{SM}}{d^4x d^3k} = \left.\frac{dN_{kin}}{dt d^3x d^3k}\right|_{t=0} \tag{VI.30}$$

  setting $\omega_p(k) \to k, \mathcal{Z}_k \to 1$ to lowest order in $\mathcal{O}(\alpha_{em})$, thus obtaining the result given by eq. (IV.14) of section (IV A).

  The non-perturbative aspects of the main result of this article, namely equations (VI.22-VI.26) make manifestly clear the shortcomings of the S-matrix approach to extract the photon emission yield, including those in ref.[36]: such approach treats the plasma as a state in LTE of infinite lifetime and assumes that the photons are produced at a constant rate. These assumptions manifestly ignore the physically correct picture that if the lifetime of the QGP is infinite then photons produced will eventually thermalize. The validity of the S-matrix approach will then have to be justified on some intermediate time scale $t \ll 1/\Gamma_k$ but then finite time effects *must* be included in the full calculation and the validity of the S-matrix approach will depend on the momentum of the photon. As we have discussed above, long-wavelength photons with $k \leq 200$ Mev thermalize on time scales of the order of the expected lifetime of the QGP at RHIC and LHC $t \sim 10 - 20$ fm/c (see fig. (4)), thus the S-matrix calculation is explicitly restricted to large momentum photons and only during a time scale $t \ll 1/\Gamma_k$ which depends on the momentum. But clearly this is in manifest contradiction with the assumptions underlying the S-matrix calculation: the existence of asymptotic states and taking the infinite time limit.

  Assuming an initial state prepared in the asymptotic past not only ignores the true physical aspects of the problem, namely that quarks and gluons are partonic degrees of freedom in the incoming colliding hadrons, not *asymptotic states*, but also that however long the mean-free path of the photon, if the plasma lives long enough, these will thermalize.

- In obtaining the final result given by equations (VI.22-VI.26) we have systematically neglected terms that are truly perturbative and of higher order in an uniform expansion in α_{em}. Namely the terms that we neglected are all bound in time and manifestly of higher order in α_{em}. The final result (VI.22-VI.26) provides a resummation of the naive perturbative expansion much in the same manner as the dynamical renormalization group[31] which leads to correct kinetic equations[24].

Our main results and predictions for the yield and spectrum of the emitted photons are displayed below in figure (5).

This figure shows the yield obtained from the result eqs. (VI.22-VI.26) as compared to the S-matrix yield for $T = 300$ Mev; $t = 10$ fm/c which are values of the temperature and lifetime expected at RHIC. We have displayed the results only up to $k \sim 5$ Gev since as discussed in [15] and above, for much larger momenta the wavelength of the

photon is likely probing scales shorter than the mean free path and the approximation of LTE is no longer reliable. Furthermore, as discussed in detail in [15], for large momenta $k \geq 4 - 5$ Gev the spectrum of photons is sensitive to the initial pre-equilibrium stage[13].

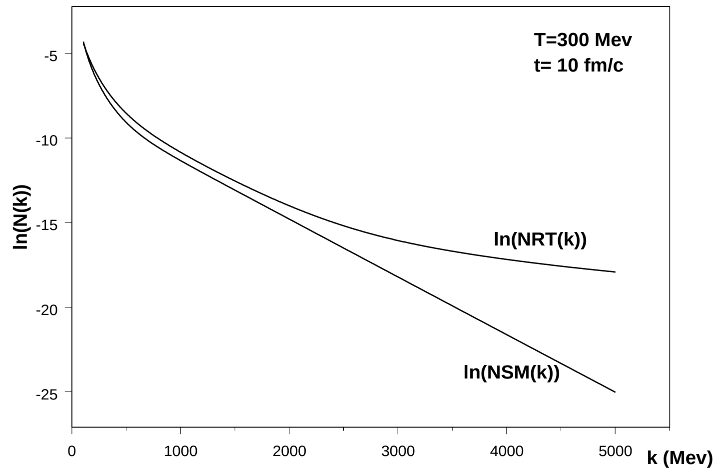

FIG. 5: $\ln\left(\frac{dN}{d^3kd^3x}\right)$ vs. k(Mev) for $T = 300$ Mev and $t = 10$ fm$/c$. $NRT(k) = dN/d^3kd^3x$ given by eqs. (VI.22-VI.26) and $NSM(k) = dN_{SM}/d^3kd^3x$ given by eqs. (VI.29,VI.14).

Clearly the real time yield is systematically larger than that predicted by the S-matrix approach. A flattening of the spectrum at $k \sim 2.5 - 3$ Gev is a distinct feature that originates in processes that while subleading in the asymptotically long time limit dominate during the finite lifetime of the QGP. We have established numerically that amongst the four terms in eqs. (VI.23-VI.26), the largest contributions to the yield during the QGP lifetime ~ 10 fm$/c$ are those from eq. (VI.23) and eq. (VI.24) while the contributions from eq.(VI.25), (VI.26) are numerically smaller for the range of momenta displayed in fig. (5). The first term given by (VI.23) features the fastest fall off with energy, while the remaining terms (VI.24-VI.26) all contribute to the flattening of the spectrum.

VII. DISCUSSION:

A. Time evolution in URHIC:

An important aspect of the real time formulation of photon production is the realization that particle production during ultrarelativistic heavy ion collisions requires the understanding of dynamics *very different from usual collision experiments*. In usual collision experiments the colliding particles in the initial beam are the 'in' states and the reaction products are the 'out' states. In and out states are eigenstates of the non-interacting Hamiltonian with the physical masses. The overlap between these states and those created by the Heisenberg field operators out of the exact vacuum at asymptotically early and late times is determined by the *vacuum* wavefunction renormalization constant Z.

Time evolution in quantum mechanics and quantum field theory is an *initial value problem* and this is the basic ingredient in S-matrix theory approach to scattering. The S-matrix approach considers the time evolution of initial states corresponding to the 'in' states, namely eigenstates of the free Hamiltonian with the physical masses which are prepared in the infinite past. The S-matrix is the time evolution operator (in the interaction picture) from $t_i = -\infty$ up to $t_f = \infty$ and evolves these initial states up to an asymptotically large time, at which the overlap between the 'out' states, eigenstates of the free field Hamiltonian of physical mass is computed. The usual reduction formulas introduce the factor Z to account for the overlap between the states created by the full Heisenberg field operator and those of the free field creation operators of in and out fields.

In URHIC the initial states are heavy ions and the final states are mesons and baryons, not the elementary quark and gluon fields in the QCD Hamiltonian. The collision involves the deconfinement, thermalization, and subsequent

re-hadronization of partons. The correct S-matrix approach should then include *all aspects* of these dynamical processes in order to provide a consistent framework for calculation. Needless to say, this is currently an impossible task. Instead, previous approaches to photon production seek to obtain the photon yield from the *different stages independently.* The pre-equilibrium stage is studied via parton cascade models which input parton scattering cross sections computed from S-matrix theory into a semiclassical transport formulation. The thermalized QGP state is studied by assuming a stationary QGP of infinite lifetime in thermal equilibrium with no memory of the initial pre-equilibrium stage. Photons from the hadronized state are studied by assuming a 'cocktail' of mesons. The results from S-matrix computations (which necessarily *assume* an infinite lifetime) in a thermalized QGP are combined with hydrodynamics to integrate the *rate* during the finite lifetime of the QGP, manifestly ignoring that the S-matrix approach has assumed an infinite lifetime. Photon production during the hadronized stage is calculated via S-matrix with in and out states corresponding to (short lived) mesons. Finally the total spectrum of photons is obtained by adding the contributions from the different stages. It should be obvious that there are glaring shortcomings in this program which justify deeper scrutiny.

The real-time approach that we advocate is solidly based on the basic premise of quantum mechanics and quantum field theory, namely that time evolution is an initial value problem: given an initial state (or density matrix) and the Hamiltonian, time evolution is completely determined. Basic quantum mechanics states that given a density matrix at some initial time t_i, $\widehat{\rho}(t_i)$, the density matrix at *any* time t (in the Schroedinger picture) is

$$\widehat{\rho}(t) = e^{-iH(t-t_i)}\, \widehat{\rho}(t_i)\, e^{iH(t-t_i)}\,, \tag{VII.1}$$

where H is the complete Hamiltonian. In the case under consideration, H is the full Hamiltonian of QCD plus electromagnetism and is *time independent.* It is convenient to pass to the interaction picture of the non-interacting Hamiltonian H_0 in which the evolution operator is given by the identity

$$e^{-iH(t-t_i)} = e^{-iH_0\, t}\; U(t,t_i)\; e^{iH_0\, t_i} \tag{VII.2}$$

where $U(t,t_i)$ is the usual unitary time evolution operator in the interaction picture.

The expectation value of *any* arbitrary Schrödinger picture operator is

$$\langle \mathcal{O} \rangle(t) = \frac{\mathrm{Tr}\;[\mathcal{O}\widehat{\rho}(t)]}{\mathrm{Tr}\,\widehat{\rho}(t_i)} \tag{VII.3}$$

This is the same as the expectation value of the Heisenberg picture operator in the initial density matrix. Computing real time expectation values is *very different* from computing S-matrix elements. The former are in-in matrix elements as befits time evolution as an initial value problem, the latter gives a transition amplitude from a state far in the past to a state far in the future and is conceptually very different from obtaining an expectation value of an operator in a time evolved state.

It remains to define the Heisenberg number operator: the Heisenberg *field* operator for photons creates a single photon state (in or out) out of the *vacuum* with probability $\sqrt{Z}$, with Z being the on-shell vacuum wave function renormalization constant. Since the number operator is a bilineal in the field it must be divided by Z in order to ensure that it counts asymptotic photon states with weight one, this is an important ingredient of the reduction formulae. Furthermore in free field theory a simple normal ordering suffices to ensure that the number operator annihilates the (bare) vacuum. However in an interacting theory, a normal ordering constant $\mathcal{C}_k$ *must* be introduced so that the expectation value of the Heisenberg field operator vanishes in the exact vacuum. This normal ordering constant can be simply extracted from the equal time limit of the operator product expansion (OPE) of the correlation function of the photon field. The short distance expansion is completely determined by the OPE in the vacuum. Therefore subtracting the constant $\mathcal{C}_k$ is *necessary* to define a normal ordered expectation value of the Heisenberg number operator consistently with the equal time limit of the OPE of the correlation function of the (transverse) gauge field.

This is *precisely* the main point of our program, we obtain the expectation value of the photon number operator given by eq. (III.11) which includes the overlap factor Z and the normal ordering subtraction $\mathcal{C}_k$ *both* determined in the *vacuum* consistently with asymptotic theory. In this article we implemented this program by obtaining the non-equilibrium effective action for the gauge field as explained in section (II). As was explicitly shown in our previous article (see [15]), the vacuum wave function renormalization constant and the vacuum subtraction $\mathcal{C}_k$ cancel the vacuum contributions to the number operator. That this is indeed the number of photons (exact massless eigenstates of the free photon Hamiltonian) produced in the medium up to time t by the evolution is clear from the following trivial equality

$$\langle N_k \rangle(t) = \langle N_k \rangle(t_i) + \int_{t_i}^{t} \frac{d\langle N_k \rangle(t')}{dt'} \tag{VII.4}$$

If the photons do not undergo further interactions after being produced during the lifetime of the QGP, they will propagate freely towards the detector.

As has been shown explicitly in section (IV) the S-matrix (or alternatively the kinetic approach) extracts the rate $d < N_k > (t)/dt$ from Fermi's Golden rule, by taking the time interval to infinity which leads to a time independent rate, and inputs this rate in the calculation of the yield just as in eq. (VII.4) above accounting for hydrodynamical expansion.

Of course if one *knew* how to evolve an initial heavy ion state through deconfinement, thermalization and finally re-hadronization with the full QCD Hamiltonian, one could begin with an initial density matrix of heavy ions at $t_i \to -\infty$ and compute the total number of photons produced *all throughout the dynamics* up to $t \to +\infty$ by implementing (VII.3). Obviously this program is not feasible. However, the statement of formation of a thermalized QGP at a time of order ~ 1 fm$/c$ suggests that at this *initial time* t_i the density matrix is that of thermal equilibrium for the QGP. Time evolution being an initial value problem, *if* the initial density matrix is *completely* specified, then we can evolve it in time with the full *time independent* Hamiltonian H. Perturbation theory will be reliable if quarks and gluons are weakly interacting, the usual assumption for a QGT in LTE but certainly *not* during hadronization. Nevertheless, we can still obtain the number of photons produced up to any arbitrary time t from a weakly interacting QGP in LTE from eq. (VII.3) *if* we knew exactly what is the distribution of photons in the initial state. The main uncertainty in the calculation of the yield is precisely the knowledge of the photon distribution function in the initial density matrix.

Only when photons reach thermal equilibrium with the medium will their spectrum and distribution be *independent of the initial conditions*. However if photons do not equilibrate, and this is the usual assumption, the spectrum of produced photons *will* depend on the initial state. In ref.[15] we have studied suitably modeled initial states and found that the photon spectrum is sensitive to the details of the initial state for energy larger than $\sim 5-6$Gev assuming a thermalization scale $\lesssim 1$fm$/c$.

Recently a criticism has been leveled at the real time formulation for studying photon production from a transient QGP[37] that we propose. The authors in this reference claim that the unitary time evolution operator $U(t, t_i)$ in the interaction picture in eq. (VII.2) corresponds to 'switching on' the interaction at $t = t_i$ and 'switching-off' at time t_f. This is clearly incorrect as the identity Eq. (VII.2) makes manifest. Such a switch-on-off of the interaction would entail a time *dependent* Hamiltonian with an artificial time dependent coupling. The identity (VII.2) clearly states that time evolution is obtained with the full time independent Hamiltonian of QCD plus QED. This statement by the authors of ref.[37] reflects a misunderstanding of the basic fact that time evolution in quantum mechanics and quantum field theory is an *initial value problem*: a state specified at some initial time t_i is evolved in time up to an arbitrary time t_f with the unitary time evolution operator given by eq. (VII.2). As we mentioned above once the density matrix (or a pure state) is specified at *any* arbitrary time t_i it can be evolved up to another arbitrary time t with the unitary time evolution operator. Any statements about 'switching on-off' of interactions is simply a misunderstanding of this basic fact in quantum mechanics. A series of formal steps are invoked in ref.[37] to incorrectly argue that only taking $t_f \to \infty$ gives the total photon number. With this statement the authors are ignoring or glossing over a large body of work in the hydrodynamic approach to photon production from a thermalized QGP. In this approach an invariant rate, obtained assuming an infinite lifetime is input in the hydrodynamical evolution, and the total photon yield is obtained by integrating over the space-time history of the QGP, namely over a *finite lifetime*. One of our main criticisms of this approach is that the rate input in these calculations is obtained from an S-matrix computation which *assumes* an infinite lifetime, but is used during a finite lifetime.

In a plasma that expands nearly at the speed of light, if its lifetime is infinite as the authors suggest, photons will ultimately thermalize with the plasma. Physically the QGP only 'lives' for a few fm$/c$, furthermore as the plasma expands and cools the coupling grows stronger and the theory becomes strongly coupled at the hadronization phase transition. Any formulation that attempts to use perturbation theory in a long time limit is obviously flawed. All of these physical arguments point out the limitations of an S-matrix approach and the inconsistency in taking an infinite time limit.

The authors in ref.[37] completely ignore the physical arguments that apply to the experimental situation therefore either ignoring or glossing over the inconsistency in taking an infinite time interval. Nor do they address the important issue of what is the initial state when the QGP is conjectured to thermalize, in particular what is the initial photon distribution function. S-matrix theory simply takes the initial time $t_i \to -\infty$ and takes the initial states to be free field 'in' states. Clearly this is not the physical situation in a heavy ion collision and the pre-equilibrium stage is important.

In ref. [37] the discussion stops short of actually calculating the number of photons in their limit $t_f \to \infty$. Had they done so they would have found a result that diverges linearly with t_f. This is precisely the result of the S-matrix calculation where t_f is interpreted as the total 'reaction time' and the result is divided by t_f to yield the rate. At this stage the physical question that the authors did not ask themselves is what is this 'reaction time'. Obviously if these photons are produced by a transient QGP, t_f must be the lifetime of this state. Lest that the authors actually mean to follow the dynamics through the confinement and hadronization phase transition, in which case their perturbative

arguments are naive at best.

The authors also question our definition of the Heisenberg number operator that includes the vacuum wave function renormalization Z and normal ordering constant $\mathcal{C}$. As explained in the text and again explicitly in this section above, the wave function renormalization is required because the interpolating Heisenberg operator creates a single in or out photon state with amplitude $\sqrt{Z}$ out of the vacuum. This is a basic result of asymptotic theory and the factor $1/Z$ is the usual factor that accompanies all bilinear correlation functions (such as the Green's function) in the reduction formulae, this is a basic issue in renormalization and in the LSZ formulation (see for example[38]). The normal ordering constant is a simple statement that the equal time limit in the operator product expansion gives a non-vanishing expectation value in the *vacuum* and must be subtracted. This is the basic statement that normal ordering and time evolution *do not commute* in an interacting field theory. Even in free field theory the number operator is defined with a normal ordering prescription (the usual symmetrization factor 1/2). Such normal ordering must be re-defined in the interacting theory and that is precisely what $\mathcal{C}$ achieves. It is defined at zero temperature, namely in the vacuum and as explicitly shown in ref.[15] it cancels the vacuum contributions to the photon number in the interacting theory.

In summary, the criticisms leveled in ref.[37] have no credibility: i) statements such as 'switching-on' and 'off' reflect a misunderstanding of basic quantum mechanics and quantum field theory, ii) formal statements about infinite time limits etc, are only glossing over the *physical* situation under consideration, namely a transient state which undergoes a non- perturbative phase transition at a finite time, iii) the statements regarding wave function renormalization and subtractions also reflect a misunderstanding of basic renormalization issues in quantum field theory, iv) the authors do not even attempt to discuss the issue of the initial state, v) the criticisms in ref. [37] do not attempt to scrutinize the usual assumptions with their glaring inconsistencies, nor do they provide a reliable framework to study the *physical situation* at hand, vi) the non-perturbative aspects associated with the *propagation* of photons in the medium (see discussion below) are not included in ref. [37], which our formulation in terms of the real time effective action describes consistently and systematically. Hence, these criticisms not only do not help to understand (or even address) the important and very relevant physical questions but disguise these fundamental aspects of URHIC physics.

B. Non-perturbative aspects:

The formulation of photon production based on the real-time effective action presented in this article allows to understand the following *non-perturbative aspects* of propagation and relaxation.

i) Propagation: The photons produced in a QGP in LTE do not propagate as free waves in vacuum but *in a medium* with an 'index of refraction'. The propagation aspects of photons in this *medium* require that self-energy corrections to the propagator be included. A simple perturbative expansion of the propagator in terms of the photon self-energy $\Sigma(\omega,k)$ is

$$G(\omega,k) = \frac{1}{\omega^2-k^2} + \frac{1}{\omega^2-k^2}\,\Sigma(\omega,k)\,\frac{1}{\omega^2-k^2} + \cdots \tag{VII.5}$$

obviously features divergences on the photon mass shell, which in real time lead to terms that grow in time as for example featured in eq. (VI.28). A Dyson (geometric) resummation of these self-energy corrections leads to the propagator

$$G(\omega,k) = \frac{1}{\omega^2-k^2-\Sigma(\omega,k)} \tag{VII.6}$$

The poles in this resummed propagator describe the correct modes of propagation in the medium, namely the transverse plasmons and their relaxational dynamics. As usual the self-energy $\Sigma(\omega,k)$ is calculated in perturbation theory up to a given order in the coupling, but the Dyson resummed propagator eq. (VII.6) provides an *all order resummation of select diagrams*. It is precisely in this sense that the real-time effective action leads to a non-perturbative description which describes the correct propagation of photons in the medium as plasmon modes. In section VI we discussed in detail the main aspects of photon propagation gleaned from the hard-thermal loop approximation to the self-energy. The poles in the Dyson resummed propagator determine the propagating modes, their frequencies and widths. Of course the dispersion relation and widths are obtained in perturbation theory up to the order in which the self-energy has been computed. This is the usual statement in many body or field theory that in order to obtain the position of the poles (propagating modes) a Dyson resummation of the self-energy corrections must be performed. These corrections that account for the propagation of photons in the medium with the correct dispersion relation are featured in the final equations (VI.22)-(VI.26) in the wave function renormalization $\mathcal{Z}_k$, the dispersion relation $\omega_p(k)$ and the width Γ_k in the denominators.

ii) Relaxation: Our final equation for the yield of photons produced during the lifetime of the QGP (VI.22)-(VI.26) features contributions with the factor $1 - e^{-\Gamma_k t}$ with Γ_k of $\mathcal{O}(\alpha_{em})$ and up to leading logarithms in α_s. The naive perturbative expansion in α_{em} replaces these factors by $1 - e^{-\Gamma_k t} \sim \Gamma_k t$, which is the usual yield obtained from the S-matrix calculation. However, such approximation is obviously only valid for $\Gamma_k t \ll 1$ (which again makes manifest the shortcoming of the S-matrix approach). However, as studied in detail in section VI A photons of energy $\sim$ 200Mev thermalize in the QGP during its lifetime of about 10fm$/c$ and the product $\Gamma_k t$ is a sizable fraction of unity for photons up to energies $\lesssim 500 - 700$Mev. Namely the full solution given by eqs.(VI.22)-(VI.26) accounts for the partial thermalization of medium energy photons as a result of the non-perturbative Dyson resummation and is explicit in the exponentials which result from the complex poles in the Dyson resummed propagators. Naive perturbation theory certainly misses thermalization since the yield obtained from a perturbative S-matrix calculation will only feature a term of the form $\Gamma_k\, t$ and it also misses the correct propagation of photons in the medium.

VIII. CONCLUSIONS, SUMMARY AND FURTHER QUESTIONS:

In this article we have focused on studying production of photons from a QGP in LTE by implementing a non-perturbative formulation of the real-time evolution of an initial density matrix. The main ingredient is the real time effective action for the electromagnetic field *exact* to order α_{em} and in principle to all orders in α_s. The real time evolution is completely determined by the solution of a non-local stochastic Langevin equation which results in a Dyson-like resummation of the naive perturbative expansion of the photon self-energy. A quantum kinetic description emerges directly from this non-perturbative formulation.

The main result for the direct photon yield from a QGP in LTE is given by eqns. (III.19) with (III.8,II.58). We have confirmed that in a strict perturbative expansion, eq. (III.19) reproduces the S-matrix results as well as previously obtained results with initially uncorrelated states[15, 16]. Furthermore we have shown that the result (III.19) reproduces the main features of quantum kinetics for the evolution of the photon distribution function.

An explicit result for the photon self-energy to order α_{em} and up to leading logarithmic order in α_s[12] indicates that photons with momenta $k \lesssim 200$ Mev propagate as plasmon quasiparticles and *thermalize* on a time scale $t \leq 10 - 15$ fm$/c$, namely of the order of the lifetime of the QGP expected at RHIC and LHC.

To leading order in a *uniform* expansion in α_{em} and up to leading logarithmic order in α_s our main result is summarized by equations (VI.22-VI.26) and our prediction for the spectrum of photons from a QGP in LTE with a lifetime of order ~ 10 fm$/c$ is displayed in fig. (5).

In the energy range of experimental relevance which we deem theoretically trustworthy 200 Mev $\lesssim k \lesssim 4 - 5$ Gev the real time yield obtained in leading order and given by equations (VI.22-VI.26) is displayed in fig. (5). It is systematically *larger* than previous estimates based on the S-matrix approach and features a flattening of the spectrum for $k \geq 2.5 - 3$ Gev. We conclude that during the finite lifetime of a QGP in LTE expected at RHIC and LHC there are important real time processes that contribute to photon emission that are not captured by the usual approach. Furthermore, a distinct prediction resulting from our study is a flattening of the spectrum, a feature that is associated precisely with these real time processes.

We have also discussed the limit of reliability of our results and the theoretical uncertainties that are present in *any* formulation of photon emission from a QGP in LTE: i) theoretical uncertainties in the initial state and pre-equilibrium stage, these will manifest themselves in the high energy part of the spectrum[13, 15], ii) a lack of knowledge of the imaginary part of the photon self-energy for *all* values of the frequency, iii) a likely breakdown of the reliability of the assumption of LTE for large energy photons. Once these uncertainties are resolved, a knowledge of the initial state (which itself requires understanding of the pre-equilibrium stage) and of the imaginary part of the photon self-energy can be systematically input in our non-perturbative formulation.

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